Indian Journal of Engineering & Materials Sciences
Vol. 27, August 2020, pp. 901-905

Design and investigation of Hybrid 2D codes for optical CDMA systems

Manisha Bharti*
Department of Electronics and Communication Engineering, National Institute of Technology, Delhi 110 040, India

Received:26 May 2020

Hybrid 2D codes have been designed, implemented and investigated for optical code division multiple access (OCDMA) system. For analysis, the codes that have been chosen in this manuscript are algebraically constructed codes, prime codes, hybrid W/T codes and optical orthogonal codes (OOCs). To investigate the performance of codes in detail, on-off keying method has been employed on all codes and combinatorial analysis has been implemented. Hard limiting error probability (HEP) and other hit probability equations have been mentioned to perform the analysis. Based on the performance analysis and simulation results, hybrid W/T code has outperformed all other optical family codes that have been taken into consideration. Emb-M/MPHC from hybrid W/T code family has shown lower bit error rate (BER) curves for increasing number of users count. The main attributes to its superior performance in comparison to all other codes have been its better code design and properties it possess.

Keywords: Optical-CDMA, MPHC, HEP, OOC, Emb-M/MPHC

1 Introduction

Optical CDMA is considered as efficient multiple access technology for high speed optical networks, due to its huge bandwidth in terahertz, secure transmission and high speed of transmission. The biggest challenge to use OCDMA technique is to design optical codes that possess ideal value of correlation properties. One dimensional (1D) optical orthogonal code (OOC) had some limitations like less count of code words, on the other hand two dimensional (2D) OOC extended codeword count while maintaining acceptable cross and auto-correlation properties. Several 2D optical codes are designed by making use of different time-spreading and wavelength-hopping patterns.

The method of designing/constructing various codes for OCDMA system differs from family to family. By applying finite fields, code construction became very simple and easy to present. Algebraic code construction can be easily applied to various code sequences such as prime sequences, OOCs and congruence codes etc. General method for algebraic construction is usually based upon evaluating finite algebraic field over a specific prime number value[1-3].

During the earlier years of development, these codes were constructed using some unique sequence equations containing a single carrier but later two other approaches were explored by researchers for prime code: sequence codes and hop codes. Sequence codes are called special family codes and they belong to member code of 2^m prime sequence code. On the other hand, hop prime codes are constructed over many carriers and utilising 2D approach for construction schemes. From there, on constructions of much enhanced and advanced 2D prime codes possessing better code dimension characteristics and improved correlation properties; prime code family enormously increased the count of member codes[4].

Optical orthogonal codes (OOC) is one of the earliest OCDMA codes that possesses correlation properties which are considered ideal for OCDMA systems. OOC basically accounts to sequences of ‘1’ and ‘0’ having equal auto- and cross correlation values maximum up to one. This property of OOC is very much effective in suppression of multiple access interference (MAI) the system. If all cyclically shifted versions of (n,k,λ) optical orthogonal code are considered orthogonal to one another, then weight of the code remains constant with length of the code.

Code construction procedure and design scheme is different for different families. Unlike hybrid W/T code family, each family utilises a unique encoding scheme for time and wavelength domain, whereas the hybrid code family employs two different sequence or optical codes in time domain and wavelength domain for time spreading as well as wavelength hopping purposes. Therefore, the newly constructed W/T optical code inherits advantages and features of both the optical codes utilised and hence are more effective and efficient than those codes as well.

*Corresponding author (E-mail: manishabharti@nitdelhi.ac.in)

In this paper, code families considered for designing and analysis are algebraically constructed codes, OOCs, prime codes, hybrid wavelength/time codes.

2 Design and properties of codes

A concise description regarding designing of codes along with its properties for each code is described in the following section one by one.

2.1 Bipolar/unipolar codes

Bipolar/unipolar code belongs to algebraically constructed code family, as it is generated by employing finite fields or by algebraic construction scheme by utilising unipolar version of bipolar code. This W/T code was generated by permutation of multi-wavelength code words onto non-zero time slots of time spreading code words which was further generated with the help of prime sequences in Galois field. To keep cross correlation constraint at most one, non-prime sequences which have repeated elements are neglected. By mapping p group of sequences with multi wavelength code words, sequence can be used as unipolar code in time and wavelength domain[5].
This bipolar/unipolar code has code length

$$n = w(w-1)\emptyset_{unipolar} + 1 \quad \dots (1)$$

Cardinality:

$$\emptyset_{Bipol/Unipol} = \emptyset_{group}\emptyset_{bipolar}\emptyset_{unipolar} \quad \dots (2)$$

Where $\emptyset_{group} = \emptyset_{bipolar} = p$ & $\lambda_c = 1$ … (3)

2.2 Hard-limiting error probability (HEP)

To analyse performance of these optical codes on-off keying (OOK) method was used and other effects like shot noise, thermal noise, background-noise, beat noise all were neglected in order to consider only effect of MAI. Combinatorial analysis was used to evaluate HEP of optical codes by employing OOK. The HEP equation is given as follows:

$$P_e = \frac{1}{2}\sum_{i=0}^{w}(-1)^l \binom{w}{l}\left(1-\frac{l\overline{q}}{w}\right)^{k-1} \quad \dots (4)$$

where q is average hit probability that is contributed by q_0 & q_i that depicts number of 1-hits from interfering code words, K refers to number of users simultaneously accessing the system. $\frac{1}{2}$ depicts equal probability of occurrence of data bits '1' & '0'.

$$q_0 = \frac{w^2(\emptyset_{unipolar}p-1)}{2n(\emptyset_{unipolar}p^2-1)} \quad \dots (5)$$

$$q_i = \frac{w^2(\emptyset_{unipolar}p-1)+(w-1)^2}{2n(\emptyset_{unipolar}p^2-1)} \quad \dots (6)$$

That yields $\overline{q} = \frac{1}{p}q_0 + \frac{p-1}{p}q_i$ … (7)

2.3 Modified prime hop code

To mitigate the adverse effect of less size of code and poor properties of correlation function possessed by these codes, a need of advanced optical prime code was felt and then prime-hop code (PHC) family were developed. PHC code family overcomes all those short comings. Modified prime-hop code (MPHC) is a member of PHC family and an extension of it as well but MPHC code design showed greater complexity. The design procedure of this code initiates with generation of modified-prime sequences followed by design of time-spreading pattern using mod. In corporating these wavelength hopping patterns with the time spreading sequences, MPHC codes are generated[6].

Code dimension: $p \times p^2, \frac{p+1}{2}, 0,1$

here p denotes number of available wavelengths; length of code (n) is given by p^2 ; value of $\lambda_a = 0$ and $\lambda_c = 1$.

Cardinality: This code has $(p-1)$number of wavelength-hopping pattern and p number of time-spreading pattern which in turn yields $p\,(p-1)$code words.

2.4 Hard-limiting error probability

$$P_e = \frac{1}{2}\sum_{i=(P+1)/2}^{N-1}\binom{N-1}{i}\left(\frac{\overline{p}}{2L}\right)^i\left(1-\frac{\overline{p}}{2L}\right)^{N-1-i} \quad \dots (8)$$

here $\overline{p}$ represents a unique value for specific prime numb end it denotes average value of hit[6]. N stands for number of active users at particular instant of time.

2.5 Extended prime code/optical code

This class of 2-dimensional code was developed by making use of extended prime code (EPC) in time domain and for wavelength-hopping, optical code sequence (OCS) was employed. Design of this code initiates with collection of prime numbers[7] as follows:

$$S_i = \left\{s_{i,o}, s_{i,1}, s_{i,2} \dots\dots s_{i,(p-1)/2}\right\} \quad \dots (9)$$

where, $s_{i,j} = 2ijmod(p)$

The sequence generated was then mapped into binary to produce EPC code. For designing, two main properties of optical code sequences (OCS) are used: First is that the sequence should not repeat itself; second each sequence should possess same length so that each frequency in the code can exist only for once.

Dimension of Code: $(m \times p^2, (p+1)/2, 0, 1)$

here m depicts wavelengths available; code length $= p^2$; weight of code $= (p+1)/2$; Maximum auto-correlation $\lambda_a = 0$ and cross-correlation $\lambda_c = 1$.

Hit probabilities:

$$Y = \frac{(p-1)\times(p+1)}{4} + \frac{(p+1)\times(p-1)}{2} + \frac{(p-1)\times(p-1)\times(p+1)}{4} + \frac{(p+2)\times(p-1)\times(p+1)}{4} \quad \ldots (10)$$

$$\mu = \frac{(p+1)\times(p+2)\times(p-1)}{(pm-1)\times 8p^2} \quad \ldots (11)$$

2.6 Hard-limiting error probability

Hard limiting error probability EPC/OCS can be described as follows:

$$P_e \le \frac{1}{2}\sum_{i=(p+1)/2}^{l-1} \binom{l-1}{i} (\mu^i)(1-\mu)^{l-1-i} \quad \ldots (12)$$

here, code weight is equivalent to the decision threshold of code and K represents total number of active subscribers using the system simultaneously.

2.7 Proposed Emb-M /MPHC

In this section, hybrid W/T code embedded m-sequence was employed for wavelength-hopping while MPHC code was used for time-spreading and this newly designed code can be referred to as Embedded-M/Modified Prime-hop code. Construction of this code starts with construction of embedded-m sequence given below[8]:

$$W_i = \{w_{i,0}, w_{i,1}, w_{i,2}, \ldots\ldots, w_{i,p-1,}\} \quad \ldots (13)$$

This is wavelength group hopping sequence over which Modified prime sequence is mapped.

Further, Emb-M sequence is generated using:

$$c_{x,y,k} = \{\Delta\lambda_g{}^{(a_{y,j}+1)} \text{if} k = a_{x,j} + j.p \text{ and}$$

$$0 \text{ for otherwise} \quad \ldots (14)$$

here, $x = 0,1,\ldots\ldots,p^2-1, y = p, p+1, \ldots . p^2 + 1$ & $j = 0,1,\ldots p-1$

Code dimension: $(p^2, w, 1, 0)$

here length of code $= p^2$; weight of code w; value of auto-correlation $\lambda_a = 1$ and value of cross-correlation $\lambda_c = 0$.

2.8 Hard-limiting error probability

BER analysis of merged m-sequence/prime code is done by combinatorial analysis.

$$P_e = \frac{1}{2}\sum_{l=Th=p}^{K-1} \binom{K-1}{l} (q)^l (1-q)^{K-1-l} \quad \ldots (15)$$

where K is total active users and p is prime number chosen

$$\text{and } q = \frac{1}{2} \times \frac{w\times(p^2-p-1)}{p^2\times(p^2-p)N-1} \quad \ldots (16)$$

here q is average hit probability.

3 Comparative analysis of existing optical codes with proposed code

In this section, comparative analysis of proposed code is demonstrated with respect to the existing codes on the basis of hard error probability as performance metric.

3.1 Performance analysis of Emb-M/MPHC code and Bipolar/Unipolar code

Figure 1 demonstrates the analysis between proposed Emb-M/MPHC code with existing bipolar/unipolar codes for optical CDMA system. The analysis curve is drawn between bit error probability rate versus number of simultaneous users accessing the system for both codes at variable parameters. From Fig. 1, it is clear that proposed Emb-M/MPHC code performed better than bipolar/unipolar codes. Bipolar/unipolar codes had larger number of wavelengths but they possess less code weight which in turn results in reduction of number of hits. At the same time, better correlation properties of this code greatly contribute to MAI mitigation. Emb-M/MPHC

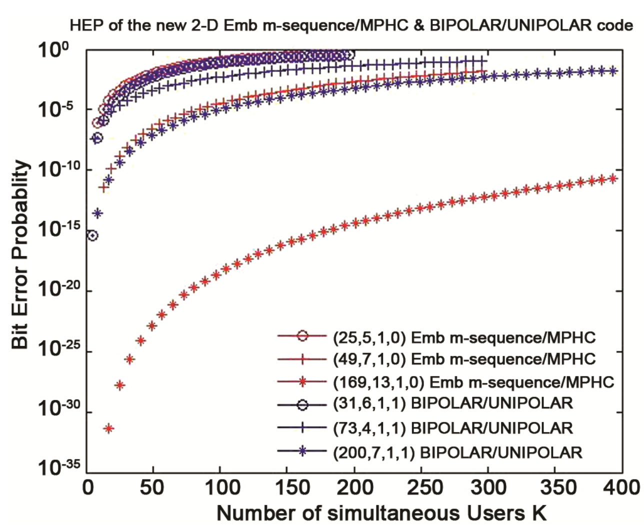

Fig. 1 — Bipolar/unipolar versus new Emb-M/MPHC.

performed better than bipolar/unipolar code and hence shows promising curve characteristics by giving low BER value even with increasing MAI as compared to bipolar/unipolar.

3.2 Performance analysis of Emb-M/MPHC code and EPC/OCS

Figure 2 shows the comparative analysis of Emb-M/MPHC code with EPC/OCS in form of bit error probability versus number of simultaneous users. Investigations reveal that proposed code performs way better than EPC/OCS codes. EPC/OCS codes had less weight therefore; number of hits produced by these codes was very less. On the other hand, large code weight of proposed Emb-M/MPHC code helped to produce more number of hits in the system and it also enhanced the performance of system by reducing multiple access interference. Emb-M/MPHC performed better than EPC/OCS and hence shows much stronger curve characteristics by giving low value of bit error probability even with increasing MAI as compared to EPC/OCS.

3.3 Performance analysis of Emb-M/MPHC code and MPHC

Figure 3 depicts the comparative performance analysis of proposed Emb-M/MPHC code with the existing MPHC code. Results clarifies that Emb-M/MPHC code also surpassed MPHC codes in view of the performance characteristics. As MPHC code was used in time-spreading of Emb-M/MPHC hence possessed all advantages of MPHC and therefore even if MPHC codes had larger number of wavelengths, Emb-M/MPHC had heavier code weight that actually compensates the number of hits; again with better correlation properties of Emb-M/MPHC multiple access interference can be reduced to a great extent. Emb-M/MPHC performed better than MPHC and hence provides lower value of bit error rate even with increasing number of users as compared to MPHC. At high value of MAI, proposed code gives better performance.

3.4 Performance analysis of EPC/OOC code and MPHC

Figure 4 proves that EPC/OCS code excels in terms of performance than MPHC codes. The BER curves are very much close to other because of EPC/OCS codes have larger number of wavelengths but have equal code weight.As almost similar correlation properties of both codes, a performance characteristic almost follows each other. But at the end only difference of number of wavelength dominates performance of both codes and hence EPC/OCS performed better than MPHC and hence shows much stronger curve characteristics by giving low BER value even with increasing MAI as compared to MPHC.

3.5 Performance analysis of EPC/OOC code and bipolar/unipolar

Figure 5 clearly shows that bipolar/unipolar code show superior performance to EPC/OCS code. Bipolar/unipolar codes had larger number of

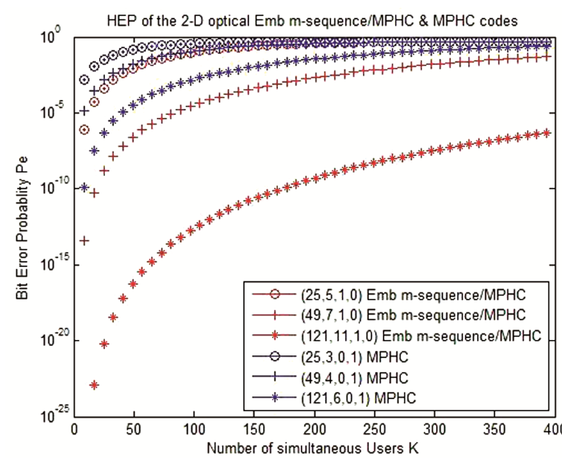

Fig. 3 — MPHC versus Emb-M/MPHC.

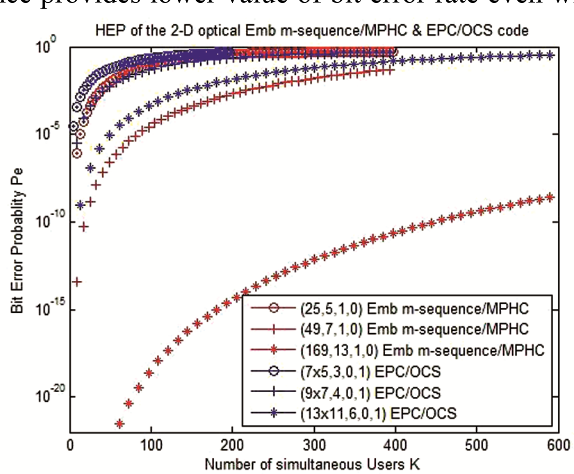

Fig. 2 — EPC/OCS versus new Emb-M/MPHC.

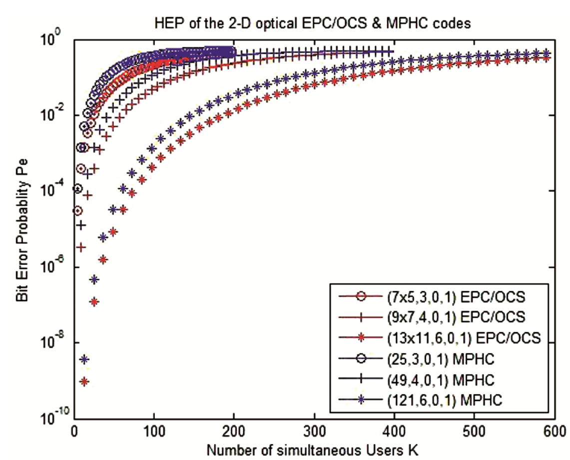

Fig. 4 — MPHC and EPC/OCS.

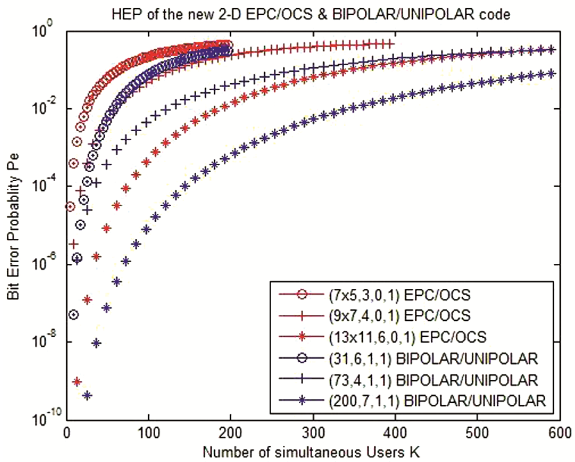

Fig. 5 — EPC/OCS versusbipolar/unipolar.

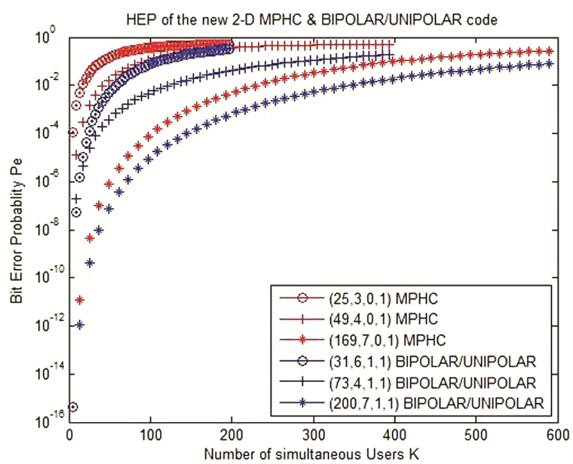

Fig. 6 — Bipolar/unipolar versus MPHC.

wavelengths and larger code weights that further results in reducing the multiple access interference. Multiple access interference is the phenomenon that occurs due to multiple users in a system accessing the network simultaneously. It generally tends to deteriorate the performance of any communication system. Due to much stronger curve characteristics by giving low BER value even with increasing MAI as compared to EPC/OCS code, bipolar/unipolar code can be concluded better.

3.6 Performance analysis of MPHC and bipolar/unipolar

Performance of MPHC code was analyzed in comparison to bipolar/unipolar code is shown in Fig. 6. Results reveal that bipolar/unipolar code outperforms MPHC code due to large number of wavelengths and high weight of code. High weight refers to more number of 1s in the code which further signifies the largest number of time slots in the given code that tend to enhance its performance. Due to best correlation properties of this code, performance characteristics of bipolar/unipolar code were enhanced and hence performed better. Moreover, the observations were taken under the effect of multiple access interference that further signified the importance of this code to use under the effect of MAI.

4 Conclusions

A detailed comparative investigation of performance characteristics of various 2-D optical codes belonging to their repsective optical code families has been done. HEP equations alongwith hit probability equations of each optical code are simulated to obtain HEP vs Number of users curves which is taken into consideration as performance characteristics.Among four distinct 2-D optical codes from algebraically construcuted code family, PC family, OOC family, Hybrid W/T code family are taken into consideration and out of these four one code shows outstanding peformance as compared to others. Performance curvesfor Emb-M/MPHC of hybrid W/T code family clearly displays lower values of BER with increasing system subscriber count as compared to other optical codes from their respective code families. Emb-M/MPHC inherits advatages and features of Embedded M-sequence code and MPHC code and thereby showing promising results. This proves a general fact that Hybrid W/T codes are much efficient and superior in performance in comparison to other code families.

References

1 Salehi J A, *IEEE Transac Comm,* 37(1989) 824.
2 Yang G C & Kwong W C, Prime Codes With Applications to CDMA Optical and Wireless Networks (Artech House), ISBN: 9781580530736, 2012.
3 Prucnal P R, Optical Code Division Multiple Access: Fundamental and Applications (Taylor & Francis), ISBN: 9781315221113, 2016.
4 Kwong W C & Yang G C, Optical Coding Theory with Prime (CRC Press), ISBN: 9781466567801,2013
5 KwongW C, Yang G C & Liu Y C, *IEEE JSelected Areas in Comm,* 23.8 (2018)1564.
6 Wen J H, Lin J Y & Liu C Y, *IEEE Proc-Comm,*150.5 (2003) 404.
7 Ji & Hua J, *Acta Photonica Sinica,* 36.7 (2015) 1285.
8 Chang & Tang Y, *Optical Eng,* 50.5 (2019) 055001.