Modelling and optimization of injection molding process for PBT/PET parts using modified particle swarm algorithm

Sagar Kumar[a], Amit Kumar Singh[a] & Vimal Kumar Pathak[b*]

[a]Department of Mechanical Engineering, Malaviya National Institute of Technology Jaipur, Rajasthan – 302017, India

[b]Department of Mechanical Engineering, Manipal University Jaipur, Jaipur, Rajasthan – 303007, India

In the present study, a systematic methodology has been presented to determine optimal injection molding conditions for minimizing warpage and shrinkage in a thin wall relay part using modified particle swarm algorithm (MPSO). Polybutylene terephthalate (PBT) and polyethylene terephthalate (PET) have been injected in thin wall relay component under different processing parameters: melt temperature, packing pressure and packing time. Further, Taguchi's L_9 (3^2) orthogonal array has been used for conducting simulation analysis to consider the interaction effects of the above parameters. A predictive mathematical model for shrinkage and warpage has been developed in terms of the above process parameters using regression model. ANOVA analysis has been performed to establish statistical significance among the injection molding parameters and the developed model. The developed model has been further optimized using a newly developed modified particle swarm optimization (MPSO) algorithm and the process parameters values have been obtained for minimized shrinkage and warpage. Furthermore, the predicted values of the shrinkage and warpage using MPSO algorithm have been reduced by approximately 30% as compared to the initial simulation values making more adequate parts.

1 Introduction

The plastic injection molding (PIM) process is widely used for producing intricate shaped plastic parts with distinctive geometric features and also has short production cycles. Basically, PIM process is a cyclic one which consists of filling and packing, cooling and ejection[1]. Injection molding helps in producing products of computer, communication, and consumer electronic (3C) such as portable computers and cell phones. The 3C products are generally thin, light, short and small[2-3]. However, with the increasing demand of more complex products having less wall thickness, the PIM process is prone to face more challenging tasks[4]. Consequently, the quality of the parts produced using PIM process is highly affected by the appropriate selection of the various process parameters and the mold design[5-6]. In contrast, the inappropriate process parameter values can lead to produce part defects, result in long lead times and high cost[7].

Warpage and shrinkage are among the most important defects that are used to measure the quality of any injection molded components. Tang *et al.*[8] applied Taguchi method for minimizing the warpage in the design of improved plastic injection mold. Similarly, Taguchi and ANOVA are used in a study for obtaining optimal shrinkage injection molding conditions[9]. The results suggest that optimized parameters reduce shrinkage by 1.244 % and 0.937 % for Polypropylene (PP) and polystyrene (PS), respectively.

Similarly, Park and Dang[10] in their study suggests that runner and cooling channel geometry can improve the final quality of products. One specific study was found for minimizing the warpage in thin shell plastic parts by employing the response surface methodology and genetic algorithm. Liao *et al.*[11] in their study provided optimum conditions for minimizing shrinkage and warpage problems. The cyclone scanner and Polyworks software was used for determining the shrinkage and warpage problem. The packing pressure was found to be the most influencing factors. Moreover, several studies found have used response surface methodology (RSM) individually or integrating with genetic algorithm (GA) for determining the interaction and relationship among factors and process parameters[12-14]. Similar studies have been found that uses neural model and modified complex method[15], grey-fuzzy logic for thin shell feature[16] for optimization of warpage in different thermoplastic parts.

*Corresponding author (E-mail: vimalpthk@gmail.com)

Generally, the warpage problem of thin-wall injection molded plastic parts has been reported in many literature. As the thickness of part decreases, the shrinkage defect becomes more complex and causes significant warpage of the plastic component. However, few of them had shown the effective processing variables and their optimization for the dimensional shrinkage and warpage minimization under high-speed injection molding process. The past literature has used only GA for the determination of optimal injection molding conditions. The current study takes into consideration a newly developed modified particle swarm optimization (MPSO) algorithm for optimal injection molding process parameter determination.

In this study, a systematic methodology is presented using regression analysis and determining the optimum process parameters using a newly developed MPSO algorithm[17]. Particle swarm optimization (PSO)algorithm has been found to be widely used as a powerful tool for solving optimization problems due to the simplicity of the concept with fewer parametric settings as compared to other population-based optimization algorithms[18-19]. However, classical PSO still has some disadvantages, such as weak local search that may lead to entrapment in local minima affecting the convergence performance that results in uncertainties in the outcomes obtained.

Due to this reason several modified variants of PSO have been proposed till date by researchers. Chen and Zhao presents a simplified PSO based on stochastic inertia weight. This variant removes the velocity parameter and obtains the inertia weight by random distribution to enhance the global and local search abilities of PSO algorithm[20]. Alfi et al. proposed an improved fuzzy particle swarm optimization (IFPSO) that utilizes fuzzy inertia weight to balance the global and local exploitation abilities[21].Verma et al. presents an opposition based modified algorithm that generates the initial particles using opposition based learning and a novel dimension based learning approach is used for finding global optimal solution[22].Tian introduced a Gaussian mutation operator to induce particle search diversity[23].A similar work by Ruan et al.[24] exploited population density to estimate the particle's distribution in the search space by introducing the swarm size, the size of the solution space and a saturated population density respectively. A constriction factor particle swarm optimization (CFPSO) is presented by Pathak and Singh, addition

of constriction factor helps in accelerating the convergence property of PSO algorithm[25]. To improve the exploitation capability of PSO, a modified particle swarm optimization (MPSO) is proposed by Pathak et al.[17] based on the generation of new improved particle position using the difference in the global and local best position. An efficient greedy selection procedure has been employed for obtaining better position between the newly generated and the current candidate solution based on the fitness value.

Furthermore, a real life case study of an electronics component relay is considered for injection molding process. The PBT/PET molten material is used to inject into the mold to produce the required constituent. A systematic methodology is proposed to analyze the volumetric shrinkage and warpage in an injection-molded part with a thin shell feature during the injection molding process. Initially, the effects of the injection process parameters on shrinkage and warpage for various wall thicknesses were examined using Taguchi method. The shrinkage and warpage values were found by moldflow insight software. The results of conducting confirmation experiments and analysis of variance (ANOVA) assure that the quadratic models of the shrinkage and warpage are well fitted with the simulated values at the optimum value. The shrinkage and warpage have been analyzed and predicted by the obtained mathematical models for the individual effects of all parameters.

2 Simulation Details

An electronic relay part, with an overall dimensions of 72 mm× 34 mm × 48.5 mm was designed in CREO-3.0 software and was used as a model, using a 3D mesh type. The analytical model consists of 29, 812 elements. The analysis were performed using three thickness values of 0.8, 0.9 and 1.0 mm. Figure 1 shows the mesh file of relay component with the cooling channels.
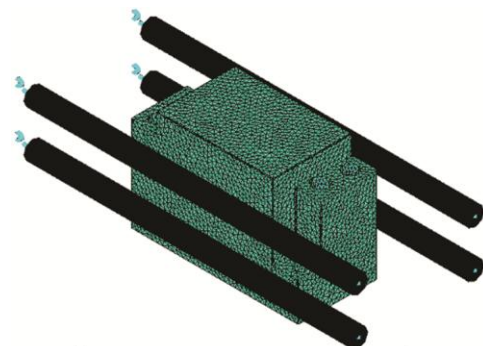

Fig. 1 — Electronic relay mesh file with cooling channel.

The default values were determined using an injection molding machine (maximum machine injection rate, 6000 m³/s) and acrylonitrile butadiene styrene (ABS) mold material (mold density, 1.5 g/cm³; mold specific heat, 1300 J/kg °C). The injection molded electronic relay was either made of PBT or PET material, due to wide application of these materials in academic and industries. The physical properties of these materials are summarized in Table 1, which was obtained from mold flow insight library. The outline of the adopted methodology for investigating optimum shrinkage and warpage prediction is shown in Fig. 2.

The parameters considered for shrinkage and warpage analyses are melt temperature (A), packing pressure (B) and packing time (C). The values of these parameters are provided in Table 2. The injection time was fixed at 3s for all experiments. An L_9 (3^3) orthogonal array was selected for the experimental design for each of the three factors. The three levels for the three factors were identified during the 9 experiments (see Table 3). The signal-to-noise ratios (S/N) for every trials were determined using:

$$\frac{S}{N} = -10 log \left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \qquad \dots (1)$$

where, n is the number of shrinkage and warpage data sets (equal to 9) and y_i is the shrinkage and warpage value for the ith data sets. The ANOVA analysis is performed using the Minitab software.

3 Results and Discussion

The Taguchi method was applied to predict the effect of injection molding process parameters on the shrinkage and warpage. The measured values of shrinkage and warpage and the signal-to-noise ratios are measured and reported in Table 4. The signal-to-noise ratio is an important quality indicator that researchers used to determine the influence of varying a particular parameter on the performance. For the current study, smaller the better characteristic was
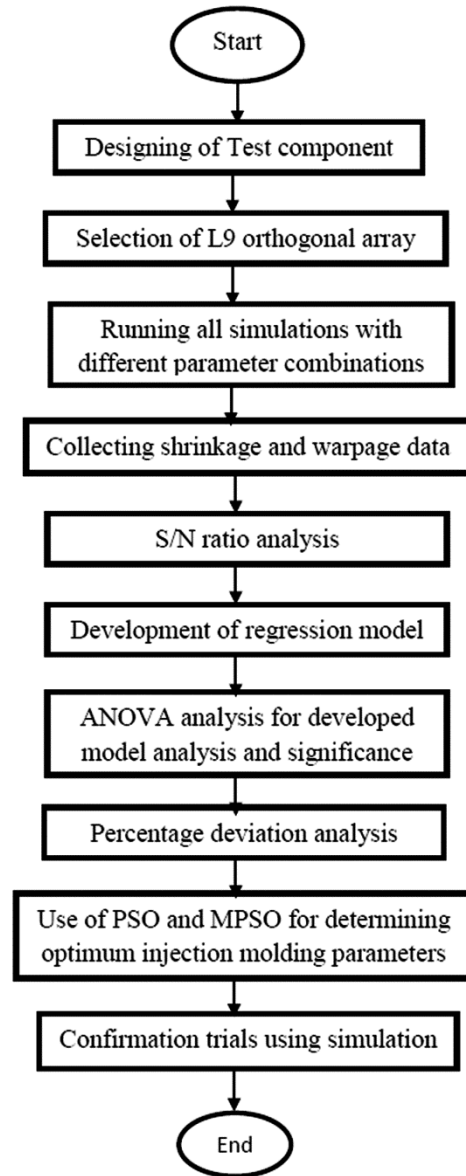

Fig. 2 — Adopted methodology for warpage and shrinkage optimization.

Table 1 — Physical properties of PBT/ PET material.

Property	PBT	PET
Melt density (g/cm³)	1.31	0.72 - 0.76
Solid density (g/cm³)	-	0.86 - 0.96
Mold temperature (°C)	40 – 60	80 – 120
Melt temperature (°C)	220 – 280	265 - 280
Poisson ratio	0.3902	035 – 0.45
Material structure	Semi-crystalline	Amorphous

Table 2 — Process Parameters and their levels.

Factors	Description	Coded Symbol	Unit	PBT Levels			PET Levels		
				1	2	3	1	2	3
A	Melt Temperature, t_m	X_1	°C	254	266	278	252	264	276
B	Packing Pressure, p_p	X_2	MPa	25	30	35	22	28	34
C	Packing Time, p_t	X_3	S	10	15	20	12	18	24

selected when calculating the S/N ratio, which is based on Eq.(1), as shown in Table 4. It was found that the best process parameter value can be determined by selecting the level with the best values at each factor. Figure 3 illustrates the variation of the S/N ratio of warpage and shrinkage for different PBT and PET material. Based on shrinkage results in Table 4 and Fig. 3 (a), it was found that PET material had higher S/N ratio (-6.02) value than the PBT material, because it exhibits least shrinkage as compared to PBT. Similarly, the warpage result shows that PET also has least warpage in comparison to PBT warpage values. It was clearly seen from the results that PET material is better for producing relay component in comparison to PBT material due to its least shrinkage and warpage.

For further analyzing the obtained results and determining the significance of each parameter, regression analysis and ANOVA test was performed. These analyses were performed on Minitab V14 software. Due to change in response statistics drastically with the control parameters, it is very challenging to develop an analytical model. The regression analysis may be the solution to this problem which is useful in searching the effect of factors to an event while examining that event. There may be factors which are either direct or indirect. The regression analysis is worthwhile when the focus is on determining the relationship between dependent and one or more independent variables. While using multiple regression analysis, the Eq. (2) of the form given below is used to explain the relationship between the independent variables X_1, X_2 and X_3 and the response variable Y.

$$Y = \beta + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 \ \dots (2)$$

Based on the above generalized regression model, the following analytical model was obtained for shrinkage of PBT and PET in the form of coded unit that can be expressed as in Eq. (3-4):

$$\text{Shrinkage (S1)} = 0.326440 + 0.201507 X_1 - 0.034872 X_2 + 0.065912 X_3 - 0.003173 X_1^2 - 0.029771 X_2^2 - 0.004712 X_3^2 + 0.033242 X_1 X_2 + 0.023560 X_1 X_3 + 0.048322 X_2 X_3 \qquad \dots (3)$$

$$\text{Shrinkage (S2)} = 0.462270 + 0.305915 X_1 - 0.048202 X_2 + 0.079210 X_3 - 0.002320 X_1^2 - 0.041725 X_2^2 - 0.003712 X_3^2 + 0.045902 X_1 X_2 + 0.043450 X_1 X_3 + 0.054721 X_2 X_3 \qquad \dots (4)$$
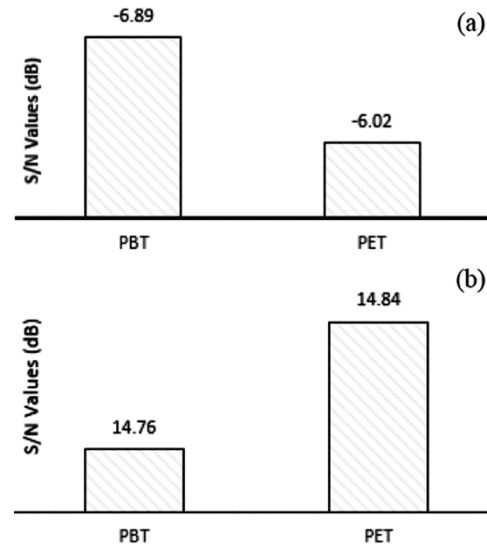

Fig. 3 — Variation of the S/N ratio of (a) Shrinkage and (b) Warpage.

Table 3 — The layout of L9 orthogonal array.

Exp. No.	A	B	C
1	1	1	3
2	1	2	2
3	1	3	1
4	2	1	2
5	2	2	1
6	2	3	3
7	3	1	1
8	3	2	3
9	3	3	2

Table 4 — Shrinkage and warpage values of PBT and PET material.

Exp. No.	PBT		PET		PBT		PET	
	Shrinkage %	S/N ratio	Shrinkage %	S/N ratio	Warpage (mm)	S/N ratio	Warpage (mm)	S/N ratio
1	2.1982	-7.1293	2.0409	-6.2237	0.2432	12.2841	0.1682	15.3867
2	2.1392	-2.3451	1.972	-6.0301	0.2164	13.2956	0.1608	15.8743
3	2.0932	-6.8343	1.9145	-5.5843	0.2813	11.0166	0.1619	15.815
4	2.1523	-7.0749	1.8832	-5.4029	0.1341	17.4521	0.2521	11.9685
5	2.0123	-6.1452	1.9923	-6.0940	0.1931	14.2844	0.2912	10.7161
6	1.9450	-5.7231	1.9639	-5.927	0.1857	14.6237	0.1972	14.1018
7	1.9197	-5.5938	2.021	-6.1512	0.2076	13.6554	0.2102	13.5473
8	1.9921	-6.0927	1.9715	-6.0211	0.1596	15.9397	0.1821	14.6143
9	1.9772	-6.0312	1.9218	-5.7439	0.1801	14.8897	0.1717	15.305

In addition, the model significance is further validated using ANOVA analysis. The ANOVA results for PBT and PET material are shown in Table 5 and Table 6, respectively. The model was built for 95% confidence level. The correlation coefficient (r^2) of the developed analytical models for PBT and PET was found to be 0.973 and 0.962 (nearer to the ideal value of 1). The adequate value of regression coefficient indicates that the model is significant and further analysis and predictions can be performed. All the linear, square and interaction terms have significant effect on the response output. It was clearly seen from Table 5 and 6 that the F-value is significantly higher for both the ANOVA table, indicating that the model is significant. There is only 0.05% chance that such a high model F-value may have occurred due to noise.

Furthermore, the performance of the developed model of warpage for both material was tested using ten randomly selected experiments other than the used in Table 4. The random values chosen from ten experiments and compared with the values obtained developed analytical model. The results of percentage deviation in prediction of shrinkage for the PBT and PET material is shown in Fig. 4. It was clearly seen from Fig. 4, the percentage deviation in shrinkage prediction for PBT and PET are 3.56 and 5.55 respectively. However, the percentage deviation for shrinkage for PBT is less as compared to PET material. Similarly, the following analytical model for warpage was developed using non-linear regression

model for both the material i.e. PBT and PET as shown in Eq. (5) and (6).

$$\text{Warpage } (W_1) = 1.23966 + 0.772017X_1 + 0.005873X_2 - 0.347592X_3 - 0.027631X_1^2 - 0.000987X_2^2 - 0.012283X_3^2 + 0.000533X_1X_2 + 0.010937X_1X_3 + 0.000272X_2X_3 \quad \dots (5)$$

$$\text{Warpage } (W_2) = 1.78218 + 0.817812X_1 + 0.007291X_2 + 0.192205X_3 - 0.051140X_1^2 - 0.001723X_2^2 - 0.027831X_3^2 + 0.000612X_1X_2 - 0.033147X_1X_3 + 0.000721X_2X_3 \quad \dots (6)$$

The developed regression model significance is further tested using ANOVA analysis. The ANOVA results for PBT and PET material are shown in Table 7 and Table 8 respectively. The model was built for 95% confidence level. The correlation coefficient (r^2) of the developed analytical models for PBT and PET was found to be 0.941 and 0.924 (nearer to the
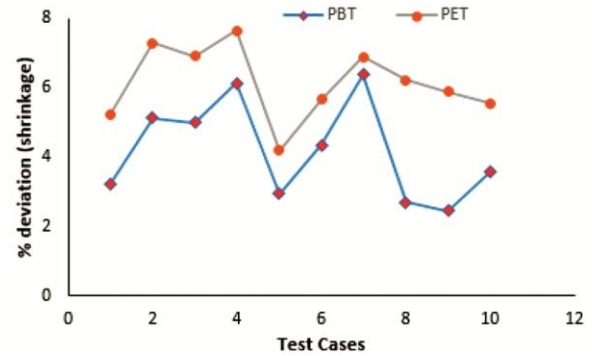

Fig. 4 — Percentage deviation in prediction of shrinkage.

Table 5 — ANOVA result for PBT shrinkage model.

Source	DF	Sum of squares (SS)	Mean square (MS)	F-Value	P-value	
Regression	9	1.52827	0.16980	253.43	0.000	Significant
Linear	3	1.37281	0.45760	682.98	0.000	
Square	3	0.01430	0.00477	7.12	0.001	
Interaction	3	0.13092	0.04031	60.16	0.000	
Residual error	41	0.02744	0.00067			
Lack of fit	5	0.02205	0.00441	6.58	0.000	
Pure error	36	0.00438	0.00012			
Total	50	1.5572				

Table 6 — ANOVA result for PET shrinkage model.

Source	DF	Sum of squares (SS)	Mean square (MS)	F-Value	P-value	
Regression	9	2.72891	0.30321	133.57	0.000	Significant
Linear	3	2.43011	0.81003	356.84	0.000	
Square	3	0.08260	0.02753	12.13	0.000	
Interaction	3	0.25109	0.08370	36.87	0.000	
Residual error	41	0.09344	0.00227			
Lack of fit	5	0.08704	0.01741	7.67	0.000	
Pure error	36	0.00812	0.00012			
Total	50	2.92391	0.00025			

Table 7 — ANOVA analysis for warpage PBT model.

Source	DF	Sum of squares (SS)	Mean square (MS)	F-Value	P-value	
Regression	9	202.317	22.48	40.43	0.000	Significant
Linear	3	112.34	37.45	67.36	0.000	
Square	3	43.22	14.41	25.92	0.000	
Interaction	3	29.23	9.74	17.52	0.000	
Residual error	41	22.78	0.556			
Lack of fit	5	16.26	3.252	5.56	0.000	
Pure error	36	6.52	0.181			
Total	50	225.097				

Table 8 — ANOVA Analysis for warpage PET model.

Source	DF	Sum of squares (SS)	Mean square (MS)	F-Value	P-value	
Regression	9	195.794	21.755	49.55	0.000	Significant
Linear	3	106.91	35.64	81.48	0.001	
Square	3	35.18	11.73	26.72	0.000	
Interaction	3	25.76	8.59	19.55	0.000	
Residual error	41	18.02	0.439			
Lack of fit	5	12.54	2.508	5.72	0.000	
Pure error	36	5.48	0.152			
Total	50	213.814				

ideal value of 1). The adequate value of regression coefficient indicates that the model is significant and further analysis and predictions can be performed. Moreover, all the linear, square and interaction terms have significant effect on the response output. It was clearly seen from Table 7 and 8 that the F-value is significantly higher for both the ANOVA table, indicating that the model is significant. There is only 0.05% chance that such a high model F-value may have occurred due to noise. Furthermore, the performance of the developed model of warpage for both material was tested using ten randomly selected experiments other than the used in Table 4. The random values chosen from ten experiments and compared with the values obtained developed analytical model. The results of percentage deviation in prediction of warpage for the PBT and PET material is shown in Fig. 5. It was clearly seen from Fig. 5, the percentage deviation in warpage prediction for PBT and PET are 3.22 and 4.23 respectively. However, the percentage deviation for warpage for PBT is less as compared to PET material.

4 Modified Particle Swarm Optimization Algorithm

This section proposes the modified variant of classical particle swarm optimization algorithm to optimize developed analytical model. Since the exploitation ability directly influences the quality of results. The modified variant will help in overcoming the classical PSO drawback of low convergence due to lack in exploitation abilities.
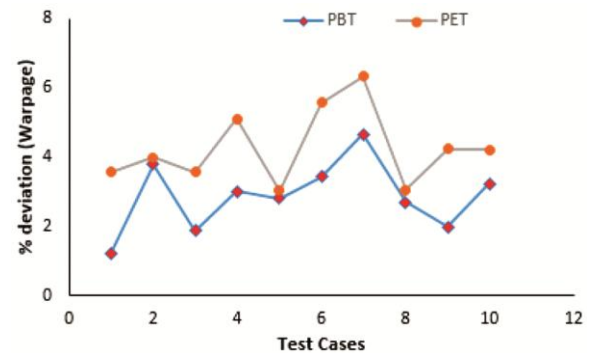

Fig. 5 — Percentage deviation in prediction of warpage.

4.1 Standard Particle Swarm Optimization Algorithm

The basic particle swarm optimization is a population-based method suggested by Kennedy and Eberhart in 1995. PSO is modeled after the simulation of social behavior of birds in a flock[26-27]. PSO is initialized by distributing each particle randomly in D-dimensional search space. The performance of each particle is measured using a fitness function which depends on the optimization problem. Each particle i represented by following information:

x_i, the current position of the particle i
v_i, current velocity of the particle i
p_i, personal best position of the particle i

The personal best position is the best position that particle i has been so far. The fitness function is highest for that position of ith particle. Here, velocity acts like a vector which helps in guiding the particle from one position to another with updated velocity at

every iteration. The personal best position of ith particle with dependence on time step t as expressed in Eq. (7):

$$P_i(t) = p_i(t) \qquad if\ f\big(x_i(t)\big) \geq f\big(p_i(t)\big)$$
$$x_i(t) \qquad if\ f(x_i(t)) < f(p_i(t)) \qquad \ldots (7)$$

New position and velocity for i^{th} particle is updated at every iteration and expressed as Eq. (8-9):

$$v_i(t+1) = v_i(t) + c_1 \qquad\qquad \ldots (8)$$
$$* r_1\big(p_i(t) - x_i(t)\big)$$
$$+ c_2 * r_2(g_i(t)$$
$$- x_i(t))$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \qquad \ldots (9)$$

r_1 and r_2 are two independent uniformly distributed random numbers within given interval [0,1]. c_1 and c_2 are two accelerating coefficients whose value are generally 2 each for almost all applications, $p(t)$ is the best position parameter of an individual particle and $g(t)$ is global best position parameter of entire swarms. Shi and Eberhart introduced an inertia weight w into the velocity updating of the PSO that helps in controlling the scope of the search. Often, w decreases linearly from 0.9 to 0.4 over the whole iteration. High value of inertia weight helps in exploration whereas low value favors exploitation. The velocity update with inertia weight is shown in Eq. (10).

$$v_i(t+1) = w(t) * v_i(t) + c_1 \qquad \ldots (10)$$
$$* r_1\big(p_i(t) - x_i(t)\big) + c_2$$
$$* r_2(g_i(t) - x_i(t))$$

4.2 Modified Particle Swarm Optimization (MPSO) Algorithm

A new variant of PSO proposed in this paper for optimizing the shrinkage and warpage in injection molded components. The exploration and exploitation capabilities are two important factors that are considered during design of an optimization algorithm. Exploitation refers to the use of existing information whereas the exploration means generation of new solution in the search space. In PSO, new solution is replaced by the old one without really comparing which one is better. This shows the lack in exploitation capability of PSO and has only exploration tendency which makes it hard to find the best possible solutions.

Because of the lack in exploitation strategy, classical PSO still have some disadvantages, such as weak local search ability and may lead to entrapment in local minimum solutions. To overcome all these

problems, the modified variant of PSO algorithm generates new swarm position and fitness solution based on the new search Eq. (11) and (12):

$$v_{new} = p_{best} + r(g_i(\text{t}) - p_i(\text{t})) \qquad \ldots (11)$$

$$x_{new} = p_i(\text{t}) + v_{new} \qquad \ldots (12)$$

where, $p(t)$ is the particle best position, $g(t)$ is the particle global best position. Parameter r denotes random number between 0 and 1 that controls the rate at which the population evolves. The best solutions in the current population are very useful sources that can be used to improve the convergence performance. Also, Eq. (11) can drive the new candidate solution only around the best solution of the previous iteration.

Therefore, the proposed search and update equations described by Eq. (11) and (12) can increase the exploitation capability of the classical PSO. Any selection strategy in the algorithm is usually considered as exploitation, as the fitness solution of the individual is used to determine whether or not an individual should be exploited. Therefore, the MPSO particle swarms employ greedy selection procedure among two parallel fitness functions to update the best candidate solution which also helps in improving the exploitation ability of the algorithm. The flowchart of proposed modified PSO algorithm is shown in Fig. 6.

4.3 Benchmark Testing

In order to prove the effectiveness of the proposed MPSO algorithm and testify its applicability in evaluating the optimal injection process parameters, five benchmark test functions are selected. These benchmark functions aim for a global minimum value and the commonly used to test any newly proposed algorithm or variant of an existing one. The test functions to be minimized include unimodal functions, multimodal functions having many local optima and multimodal function having local optima in the pre-defined search space. Results obtained using the MPSO algorithm are compared with the results of other modified variant of PSO proposed in literature. The five test functions are defined as follows:

a. Benchmark Function 1

The sphere function, f_1 is defined as follows in Eq. (13):

$$f_1 = \sum_{i=1}^{D} x_i^2 \qquad \ldots (13)$$

The function has a unique global minimum value of 0, and the search space is $-100 < x_i < 100$.

Fig. 6 — Flowchart of modified particle swarm optimization (MPSO) algorithm.

where x_i is the design variable (For $i = 1,2 .... D$ dimensions)

b. Benchmark Function 2

The Rosenbrock parabolic valley function, f_2 is defined as follows in Eq. (14):

$$f_2 = \sum_{i=1}^{D-1} 100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2 \qquad ... (14)$$

The function has a unique global minimum value of **0,** and the search space is $-2 < x_i < 2$.

where x_i is the design variable (For $i = 1,2 .... D$ dimensions)

c. Benchmark Function 3

The Powell Quartic function was proposed by MJD Powell in 1962. It is a unimodal test function which is used to test the convergence performance and optimization effectiveness of the new optimization algorithms for several variables. The Powell Quartic function, f_3 is defined as follows in Eq. (15):

$$f_3 = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2$$
$$+(x_2 - 2x_3)^4 + 10(x_1 - x_4)^4 \qquad ... (15)$$

The function has a unique global minimum value of 0 and the search space is $-4 < x_i < 5$.

where, x_i (x_1, x_2, x_3, x_4) is the design variable (For $i = 1,2,3,4$ dimensions)

d. Benchmark Function 4

It is a generic sample of non-linear multi-modal function. It was proposed by Rastrigin. Analytically, it represents very hard problem due to its large search space and it large number of local minima. The Rastrigin function, f_4 is defined as follows in Eq. (16):

$$f_4 = \sum_{i=1}^{D}[x_i^2 - 10\cos(2\pi x_i) + 10] \qquad ... (16)$$

The function has many local minima and a unique global minimum value of 0. The search space is $-5.12 < x_i < 5.12$ within which the x_i variable will search the optimum solution. This function can be used for testing the ability of new optimization algorithms in searching and escaping from the local extreme points.

where x_i is the design variable (For $i = 1,2 .... D$ dimensions)

e. Benchmark Function 5

The two-dimensional Goldstein-Price function, f_5 is defined as follows in Eq. (17):

$$f_5 = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x1x2 + 3x22·30 + 2x1 - 3x2218 - 32x1 + 12x12 + 48x2 - 36x1x2 + 27x22 \qquad ... (17)$$

The global minimum value of the function is **3** and the search space is $-2 < x_i < 2$ where $(i = 1, 2)$.

where x_i (x_1, x_2) is the design variable (For $i = 1, 2$ dimensions)

The optimization results for all the SPSO [27], IFPSO [28], OPSO [29], CFPSO [32] and MPSO algorithms are shown in Table 9. The mean and standard deviation for evaluations on all the test functions is shown in Table 9 which also exhibit the effectiveness and precision of the proposed algorithm. The average number of function evaluations reflects the convergence rate of the algorithm. For benchmark function 4 and 5, the proposed MPSO takes less number of function evaluations to find the global optimum without trapping in local minima. f_1, f_2 and f_3 are unimodal functions primarily used to test the optimization accuracy and performance of the algorithm. For multimodal functions f_4 and f_5 with more local minima points, it was found that the MPSO algorithm shows capability of escaping from local minima to provide global optimization. As clearly seen from the results in Table 9 MPSO outperforms all the selected variant of PSO in optimization accuracy, function evaluations and convergence. Therefore, proposed MPSO can be applied for effective evaluation and optimization of process parameters in plastic injection molding. Since the results of IFPSO is closest to the presented MPSO approach, the convergence of MPSO along with IFPSO is shown for the benchmark functions (see Fig. 7).

5 Optimization Problem Formulation

In the present study, a mathematical model of the shrinkage and warpage is minimized to achieve optimum values of melt temperature, packing pressure and packing time. The optimal parameters values are required to have improved injection molding process.

5.1 Identification of Design Variables

The developed regression models for shrinkage and warpage is composed of three important parameters i.e. melt temperature, packing pressure and packing time. These three parameters are taken as the design variables.

5.2 Objective Function and Constraints

For improving the final accuracy of injection molded components, the shrinkage and warpage in S_1, S_2 and W_1 and W_2 needs to be minimized. Now the optimization problem is formulated and is given as Eq. (18):

Table 9 — Comparison of simulation results for benchmark functions.

Benchmark Function	Algorithm	Best	Mean	Std. Deviation	Avg Number Func Eval
1	SPSO	3.1442E-19	2.1565E-18	7.2340E-17	50000
	IFPSO	6.4135E-22	9.237E-21	6.862E-21	50000
	OPSO	2.76836E-10	2.198E-08	4.598E-08	50000
	CFPSO	4.4912E-12	5.3321E-14	4.8890E-13	50000
	MPSO	1.1276E-32	1.007E-30	5.997E-30	50000
2	SPSO	1.92E-07	3.22E-06	0.001115	50000
	IFPSO	1.53E-10	2.79E-9	3.95E-9	50000
	OPSO	0.010742	25.33753	5.51725	50000
	CFPSO	2.2956E-6	1.7853E-3	3.9812E-5	50000
	MPSO	3.915E-27	1.582E-26	4.442E-26	50000
3	SPSO	4.28389E-07	0.002459	0.0067926	25000
	IFPSO	1.93E-9	3.79E-07	4.11E-07	25000
	OPSO	6.17E-05	7.91E-05	7.98E-05	25000
	CFPSO	7.932E-05	3.84E-04	5.433E-04	25000
	MPSO	3.77E-14	1.09E-11	8.287E-11	25000
4	SPSO	5.6318E-4	0.596975	0.9806	25000
	IFPSO	1.581E-8	6.818433	6.3046	25000
	OPSO	4.14E-4	2.87E-2	2.932E-1	25000
	CFPSO	8.653E-1	0.45391	0.8532	25000
	MPSO	0	0	0	6450
5	SPSO	3.0000	5.3875	8.4599	5000
	IFPSO	3.0074	3.3621	0.3974	1250
	OPSO	3.0123	3.5721	0.4576	1250
	CFPSO	3.0023	3.3921	0.3369	1250
	MPSO	3.0000	3.0000	2.26E-9	1000

Minimize $f(t_m, p_p, p_t) = S_1$

$f(t_m, p_p, p_t) = S_1$

$f(t_m, p_p, p_t) = W_1$ … (18)

$f(t_m, p_p, p_t) = W_1$

For solving the optimization problem, a computer code developed in Matlab R2014a for the objective functions and modified PSO implemented as the solver. The MPSO program employed different settings of PSO parameters to predict the values of the injection molded parameters and obtain minimized values of shrinkage and warpage for final components. For proving the effectiveness of the proposed MPSO algorithm, its results are compared with those obtained from the standard PSO algorithm. The parameters for both the algorithms are set as: c1, c2 = 2.05, number of population size = 10.

The results predicted for minimized shrinkage and warpage using PSO and proposed MPSO for optimized values of melt temperature, packing pressure and packing time are shown in Table 10. It can be seen from Table 10 that the predicted values of injection molded relay component by MPSO algorithm shows significant improvement over PSO results as well as the simulation results by 36.47 % for PBT shrinkage and 19.31 % for PBT warpage

respectively. Similar results are found for PET material having shrinkage reduction of 30.73% and warpage reduction of 28.79%. The convergence graph of MPSO algorithm in comparison to standard PSO for PBT material is shown in Fig. 8. It is observed from Fig. 8 that MPSO algorithm requires only 20 iterations to converge to the optimum solution as compared to the basic PSO which needs about 60 iterations for optimum solution. The low values of shrinkage and warpage confirms that the proposed MPSO algorithm provides improved results. This will enhance the final accuracy and quality of the injection molded component and hence the result of injection molding process will be improved.

6 Confirmation Simulation

In order to test the adequacy of the developed mathematical model and justify the use of newly developed MPSO algorithm, four simulation runs were performed for shrinkage (S1, S2) and warpage (W1, W2). The shrinkage and warpage results for PBT material is shown in Fig. 9 and 10, respectively for the chosen parameter values. The data from the confirmation trials and their comparison with the predicted values of shrinkage and warpage using MPSO algorithm is shown in Table 11. From Table 11,
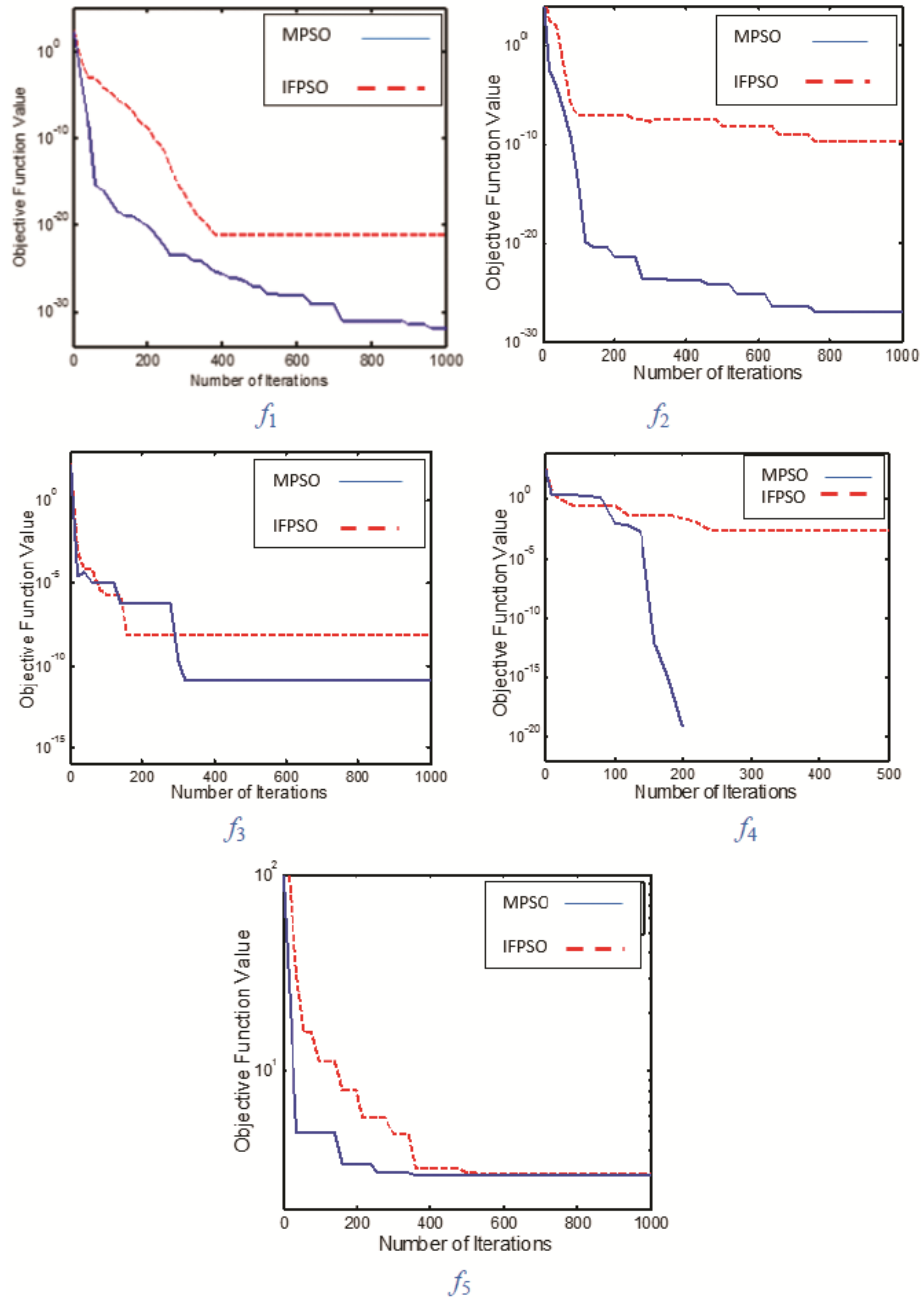
Fig. 7 — Convergence for benchmark function using IFPSO and MPSO.

Table 10 — Optimum parameters prediction using PSO and MPSO.

| Parameters	Melt temperature (T_m)		Packing pressure (P_p)		Packing time (P_t)		S1 (%)	S2 (%)	W1 (mm)	W2 (mm)
	PBT	PET	PBT	PET	PBT	PET	PBT	PET	PBT	PET
Initial value	278	252	25	34	10	12	1.9197	1.9145		
	266	252	25	28	15	18			0.1341	0.1608
PSO	279.32	254.02	27.12	29.65	10.87	12.07	1.4203	1.5182		
	269.72	253.91	26.79	29.01	14.67	17.89			0.1221	0.1365
MPSO	281.09	254.15	28.54	35.17	10.34	12.53	1.2196	1.3298		
	268.26	253.12	27.91	28.34	15.91	18.22			0.1082	0.1145
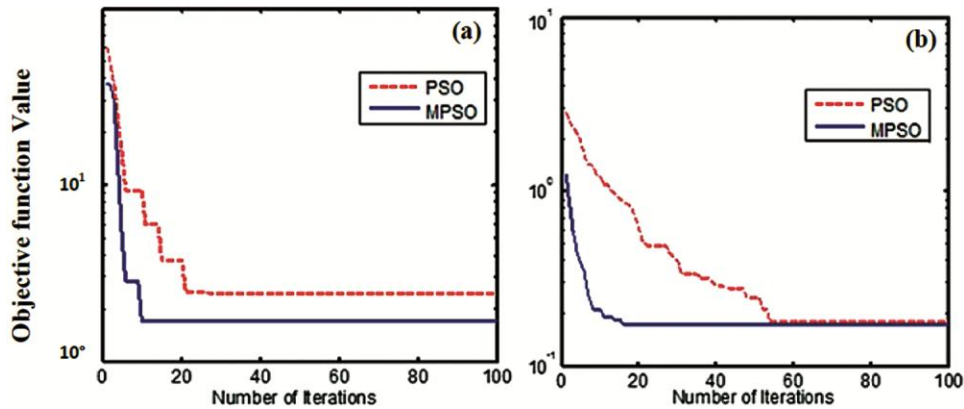
Fig. 8 — Convergence plot of (a) Shrinkage and (b) Warpage of PBT material.
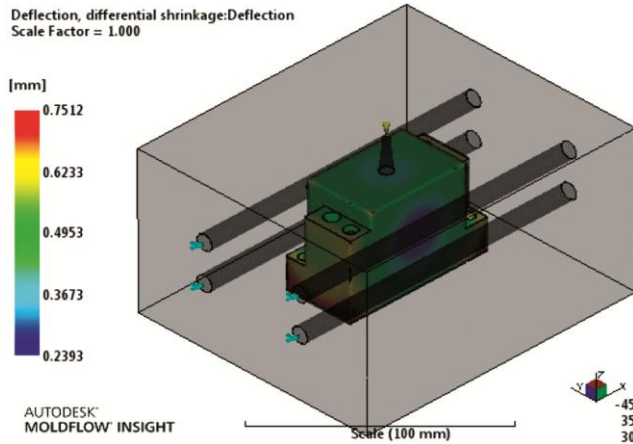

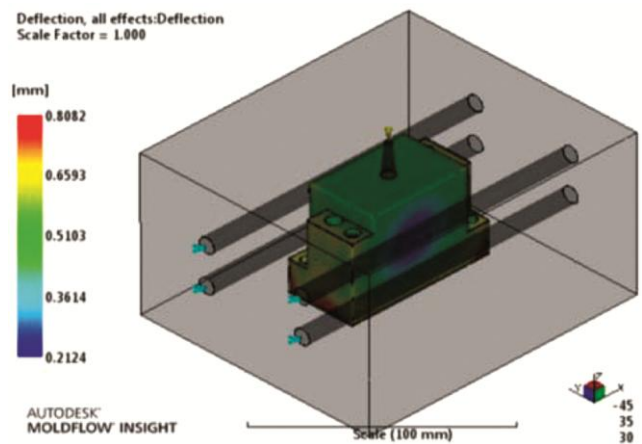Fig. 9 — Shrinkage analysis using optimized parameter.   Fig. 10 — Warpage analysis using optimized parameter.

Table 11 — Confirmation simulation trials.

S. No.	Parameters			Shrinkage (%)		Warpage (mm)	
	A	B	C	Simulation	Predicted	Simulation	Predicted
1	278	34	8	2.2289	2.2003	0.2922	0.2812
2	280	36	10	1.8722	1.7621	0.3681	0.3603
3	282	38	12	2.8021	2.745	0.2690	0.2231
4	281.09	35.17	10.34	2.321	2.1098	0.2901	0.2521

it is clearly seen that the predicted values of shrinkage and warpage are more accurate for the predicted model in comparison to the default simulation parameters.

7 Conclusions

This paper presents an integrated methodology for developing mathematical models and predicting the values of shrinkage and warpage by correlating them with process parameters of plastic injection molding process for making the electronic relay component of PBT and PET material. The parameters considered for the prediction of shrinkage and warpage are Melt temperature, Packing temperature and Packing time.

To find the optimum value of process parameters, the analytical model using regression analysis was developed. To further improve the optimum values a recently developed modified particle swarm optimization algorithm was used. The conclusions of the research are as follows:

(i)   The results of ANOVA analysis conducting confirmation experiments show that the analytical models of the shrinkage and warpage are fairly well fitted with the simulation values. The influences of all the process parameters on the performances of shrinkage and warpage have been analyzed by the obtained mathematical models.

(ii)   The predicted response of injection molding process also shows an improvement, using a newly developed MPSO algorithm. The MPSO algorithm overcomes the lack of classical PSO in exploitation

(iii) Behavior through introduction of an improved search equation based on the best solution of the previous iteration. Additionally, a greedy selection procedure is added to improve the exploitation ability of the classical PSO. The improvement in shrinkage and warpage is around 30% as compared to the initial values of shrinkage and warpage.

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