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Numerical analysis of free folding of flat textile products and proposal of new test concerning bending rigidity

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Free folding of flat textiles has been studied by means of elastica for the same shape and folding conditions across the product. Elastica is described using the system of six first-order differential equations accompanied by a set of boundary conditions. The problem is solved by the shooting method and divided in two stages. The shooting method is ineffective for some parameters, and the process is divergent which is clarified by sensitivity analysis. The proposal adopts the width of fold as a measure of bending stiffness (C) and the sample is now subjected to bending in both directions in a wide range of curvatures. This is alternative to the Peirce's test, in which the test specimen is subjected to a slight bending in one direction only. The Peirce's test gives the unprecised results under some loadings. This study presents a detailed sensitivity analysis of some process parameters to describe the unique solution for bending problem of textiles. It is also proposition for new test for bending rigidity of textiles.

Keywords: Bending length, Bending rigidity, Fabric folding, Peirce's test, Sensitivity analysis, Woven fabric

1 Introduction

The stacking or folding of the flat textile products can be analyzed using Peirce's cantilever bending theory[1]. Hearle *et al.*[2] have pointed out that textile structures can be classified into an hierarchical structure representing the deformations of one-, two- and three-dimensional continua. The properties of a particular continuum depend generally on the continuum properties of lower level units and the interconnection. These relationships have been studied for yarn structures and deformations, as well as fabric structures[3,4], but little work appears to have been done on the deformations of fabrics treated as two-dimensional continua, partly due to the difficulty in characterizing the elastic properties of textile sheets[5], and partly due to the difficulties in obtaining solutions[6].

This paper presents an analysis of the cylindrical bending of a planar fabric using both physical and mathematical models of planar bending curves and elasticas[7-10]. The flat deflection curve describes that heavy elastica is obtained under the decisive influence of gravitational force. The system of differential equations accompanied by boundary conditions describes the physical behavior of elastica. The problem is solved by the shooting method[11] and divided in two stages. Next, the sensitivity of some parameters is analyzed to clarify the divergence of the shooting method. The new test introducing the bending rigidity is proposed as the alternative to Peirce's method. The proposal adopts the width of fold as a measure of bending stiffness (*C*). Stuart *et al.*[12] and Grossberg[13] describe theoretical considerations concerning the bending of textile structures and evaluate some practical applications of fabric behavior. The bending rigidity was presented by Szablewski[14], and Grosberg and Swani[15]. The folding of flat textile fabrics was analyzed by Lloyd *et al.*[16] and Liu *et al.*[17]. The problem is solved using the sensitivity method to optimize the range of application of some decidable parameters. The sensitivity was analyzed using direct and adjoint approaches by Korycki[18] and efficiently applied in textile engineering[19,20].

The novelty of this study is (i) the dimensionless description of heavy elastica using the differential equations with a set of boundary conditions; (ii) the sensitivity analysis of parameters to describe the unique solution of shooting method; and (iii) the proposal of a new universal test for textile sample under bending in both directions (face to face, and back to back) which adopts the width of fold as a measure of bending stiffness and can be the alternative to Peirce's method.

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2 Materials and Methods

2.1 Modeling of Elastica

The elastica is defined as the one-dimensional structure of bending rigidity (C), characterized by the linear weight (q) and subjected to large deflections. Here the flat textile product is subjected to bending (e.g. woven fabric) of the specified length (l) and width (b). The shape and folding conditions are the same across the product. Thus, the space three-dimesional problem can be reduced to an optional longitudinal cross-section. The flat deflection curve, described as heavy elastica, is obtained under the decisive influence of gravity force as shown in Fig. 1(a). The inextensible structure is subjected to plane stress, i.e. the cross-sections do not influence each other.

Each point of the coordinate (s), measured along the elastica, is defined by the Cartesian coordinates $x(s)$, $y(s)$. The internal forces within the deformed elastica can be reduced to the force components $[F_x(s) \text{ and } F_y(s)]$ and the bending moment $M(s)$.

The equilibrium of an infinitesimal elastica sector (ds) helps to formulate the following system of equations, as shown in Fig. 1(b):

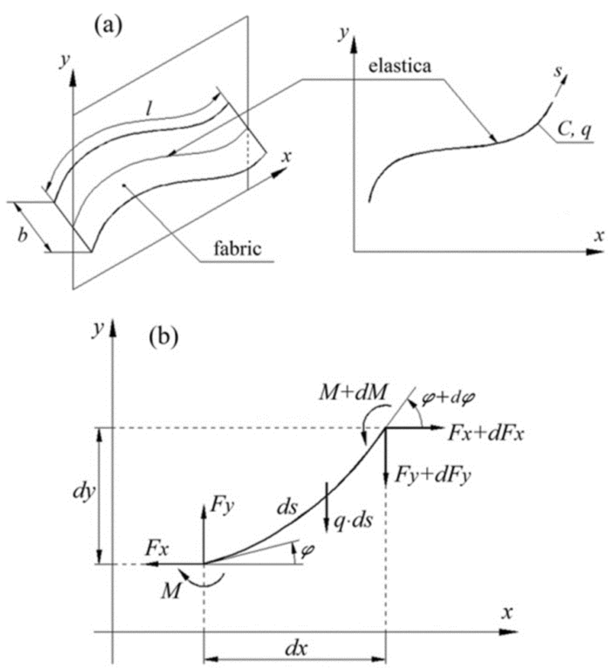

Fig. 1 — (a) Model of flat textile product approximated by elastica, and (b) equilibrium conditions within infinitesimal elastica sector

$$\begin{aligned} &-F_x + \left(F_x + dF_x\right) = 0, \\ &F_y - \left(F_y + dF_y\right) - q ds = 0, \\ &M + F_x dy + F_y dx - \left(M + dM\right) = 0. \end{aligned} \qquad \ldots (1)$$

Introducing the stable length of the sector ds, we can denote

$$\frac{dx}{ds} = \cos\varphi, \qquad \frac{dy}{ds} = \sin\varphi. \qquad \ldots (2)$$

According to Peirce, the next equation is a physical linear model as shown below:

$$M = C \cdot \kappa = C \frac{d\varphi}{ds}, \qquad \ldots (3)$$

where $\kappa = \dfrac{d\varphi}{ds}$ is the curvature; φ, the deflection angle of elastica segment; M, the bending moment; and C, the bending rigidity. Applying Eqs (1)-(3), the first-order differential equations have the following form:

$$\begin{aligned} &\frac{d\varphi}{ds} = \frac{M}{C}, \quad \frac{dM}{ds} = F_x \sin\varphi + F_y \cos\varphi, \quad \frac{dF_x}{ds} = 0, \\ &\frac{dF_y}{ds} = -q, \quad \frac{dx}{ds} = \cos\varphi, \quad \frac{dy}{ds} = \sin\varphi . \end{aligned} \qquad \ldots (4)$$

The equations of heavy elastica are generally determined by the coordinate s within the unknown parameters: (F_x, F_y, M, φ, x, y). The differential equations are nonlinear in respect of trigonometrical functions and it is necessary to define the boundary conditions at the ends of elastica. There are four principal cases of introduced constraints which allow to formulate six boundary conditions (Fig. 2).

To rearrange the equations to the dimensionless form, we introduce an additional parameter

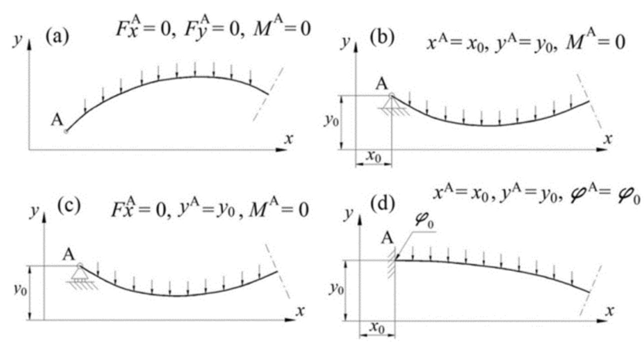

Fig. 2 — Boundary conditions of heavy elastica, where point A is (a) free, (b) supported by fixed bearing, (c) supported by mobile bearing, and (d) fixed

connecting the bending rigidity (C) and linear weight (q) in the form $D[\mathrm{m}] = \left(\frac{C}{q}\right)^{\frac{1}{3}}$, that is the bending length according to Peirce. The set of dimensionless parameters has the following form:

$$\bar{s} = \frac{s}{D},\ \bar{M} = \frac{M}{D^2 q},\ \bar{F}_x = \frac{F_x}{Dq},\ \bar{F}_y = \frac{F_y}{Dq},\ \bar{x} = \frac{x}{D},\ \bar{y} = \frac{y}{D}. \quad \ldots (5)$$

The dimensionless equations can be denoted in respect of above parameters as follows.

$$\frac{d\varphi}{d\bar{s}} = \bar{M},\quad \frac{d\bar{M}}{d\bar{s}} = \bar{F}_x \sin\varphi + \bar{F}_y \cos\varphi,\quad \frac{d\bar{F}_x}{d\bar{s}} = 0,$$
$$\frac{d\bar{F}_y}{d\bar{s}} = -1,\quad \frac{d\bar{x}}{d\bar{s}} = \cos\varphi,\quad \frac{d\bar{y}}{d\bar{s}} = \sin\varphi. \quad \ldots (6)$$

The only fundamental solution of above equations is determined for a weightless elastica ($q = 0$) by elliptic integrals[21]. Therefore, the problem is solved numerically as a set of numerical values of unknown functions at particular points of elastica.

2.2 Free Fabric Folding

The numerical shooting method is applied to solve the typical problems in textile engineering. The problem of differential equations with boundary conditions is transformed to the equivalent one with the initial conditions.

The physical model of folding effect is shown in Fig. 3(a). The woven fabric is folded on the rigid base by gravity force of the increasing length of elastica (l), leaving the feeding rollers. These rollers are located at the height (H) and have the insignificant angular velocity as constant value. The other elastica end is connected with the woven fabric.

Let us assume that the support by feeding rollers (point A) is modeled by mobile bearing to secure the correct feeding. The tangential line to elastica axis in point A should be vertical. The other end (point B) is simulated by means of fixed bearing between the fabric and the base. The bending of fabric is determined by linear model according to Peirce of the constant bending rigidity (C). Based on the observations of elastica, it can be assumed that the folding consists of two stages, as indicated in Figs 3(b) and (c).

Stage I begins with the vertical shape and $l = H$. This phase ends for the length (l) assuring the angle $\varphi = 0$ between the tangent to elastica and the horizontal axis at point B. Then the Stage II begins when the variable portion of elastica is flat and rests on the base. Stage II ends for the length (l), when the fold is completely closed [Fig. 3(c)]. The mathematical model also introduces two steps. Some solutions can be ambiguous for the growing length of elastica $l > H$, because their physical interpretation is illogical.

The folding of elastica is described using the compressive forces which is physically analogous to the buckling of bar of the prescribed length (l). The consecutive forms of critical force according to Euler equations are accompanied by the different deflection curves. The only simplest form of bending curve, equivalent to the lowest potential energy, maps the correct shape of elastica and is regarded as the correct one. The higher orders of critical forces and the corresponding deflection curves are superfluous.

The length of elastica grows stepwise and the length increment is equal to Δl. Each new length requires that the system of differential Eq. (4) should be solved again.

The solution procedure defined by the coordinate s between point B ($s = 0$) and point A ($s = l$) needs

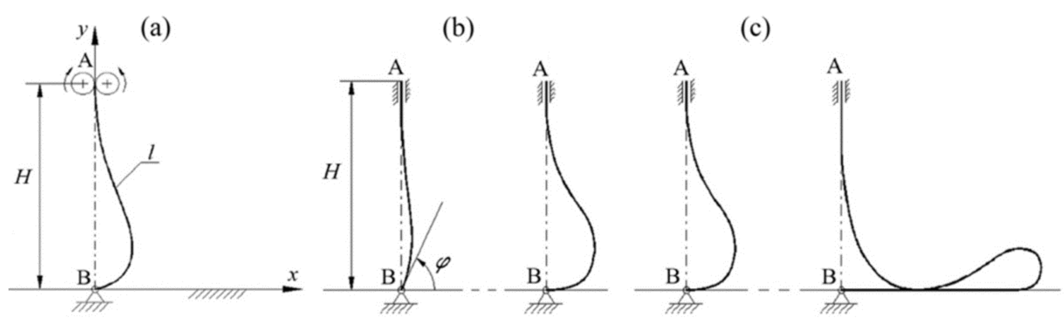

Fig. 3 — (a) Physical model of elastica in Cartesian coordinate system, (b) Stage I of elastica folding, and (c) Stage II of elastica folding

six different boundary conditions in point B to determine the variables φ, M, F_x, F_y, x and y. There is a typical boundary value problem of variable boundary conditions, depending on the folding stage. The known and unknown variables at the end points are compiled in Table 1 for the Stage I. Boundary conditions during the Stage II of folding are assumed accroding to Fig. 4(a).

Let us assume that the complete length of elastica released from point A is equal to l_2. Its part is located flat on the base from point B to point K, of the partial length equal to the unknown *x*-coordinate of a point K x_K. The curvatures of the segment BK and at the point K are equal to zero, and therefore the bending moment $M = 0$. It follows that the point K works like a fictitious bearing. Therefore, the differential Eq. (4) should be integrated in respect of *ds* from point K to point A on the length ($l_2 - x_K$), by the deflection angle $\varphi = 0$ at point K. The horizontal compressive forces are so small that, according to results published earlier[8], there is no risk of stability loss for BK segment, i.e. its rectilinear form is a stable equilibrium. The boundary conditions at the end points during the Stage II are listed in Table 2.

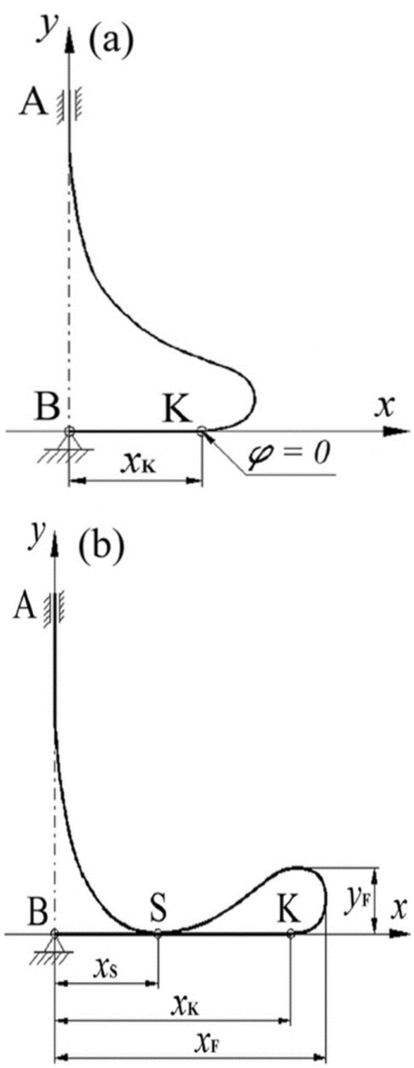

Fig. 4 — (a) Temporary shape of elastica during Stage II, and (b) parameters characterizing the elastica fold

3 Results and Discussion

3.1 Solution by Means of Shooting Method

The elastica folding is solved by means of dimensionless Eq. (6) for the assumed parameters, considering the height of rollers *H* = 1 m; the bending length *D* = 0.35 m.

Solution of Stage I

Let us assume the initial length $l = 1.01$ m. According to Table 1, three variables at point B, viz. $\overline{F}_x, \overline{F}_y$ and φ are unknown and should be assumed to complete the initial vector $\mathbf{c}^{(0)}$ of shooting method. The boundary conditions at point *A* ($\bar{s} = \frac{l}{D}$) have the following form:

$$\begin{cases} r_1 \equiv \bar{x} = 0, \\ r_2 \equiv \bar{y} - \frac{H}{D} = 0, \\ r_3 \equiv \varphi - \frac{\pi}{2} = 0\,. \end{cases} \quad \ldots (7)$$

The current values r_1, r_2, r_3 can be determined after the solution of Eq. (6) with the initial conditions. The initial-value problem is solved using the

Table 1 — Boundary conditions at the end points of elastica during the Stage I

Point	Known variables	Unknown variables
Initial point B ($s = 0$)	$x = 0, y = 0, M = 0$	F_x, F_y, φ
End point A ($s = l_1$)	$x = 0, y = H, \varphi = \pi/2$	F_x, F_y, M

Table 2 — Boundary conditions at the end points of elastica during the Stage II

Point	Known variables	Unknown variables
Initial point K ($s = 0$)	$\varphi = 0, y = 0, M = 0$	$F_x, F_y, x = x_k$
End point A ($s = l_2 - x_k$)	$x = 0, y = \text{H}, \varphi = \pi/2$	F_x, F_y, M

fourth-order Runge-Kutta method. The vector of shooting method $\mathbf{c}$ will be improved by Newton's procedure until the system of differential equations will be satisfied with the predefined accuracy ε which is the end condition in the Newton's method, namely

$$\sum_{i=1}^{3}|r_i| \le \varepsilon .$$

The proper selection of initial vector $c^{(0)}$ is very important. Introducing the prescribed length of elastica l, the vector should be as close as possible to the correct solution, i.e. the boundary conditions (7) are optimal. Otherwise, the iterative process can be divergent. The length always increases by Δl = 0.01 m. The initial vector $c^{(0)}$ is now adopted as the solution of the previous step and the calculations are iterative repeated for the new length. The stop criterion is satisfied when the angle at point B is equal to zero (φ = 0), which is at the end of Stage I. The corresponding length of elastica (l_{g1}) is called the border length of Stage I. In this example, l_{g1} = 1.336 m.

Solution of Stage II

The initial elastica length is $l = l_{g1} + \Delta l$ and it increases by a constant increment. Point K is characterized by the coordinate $\bar{s} = 0$ and conditions $\varphi = 0$, $\bar{y} = 0$, $\overline{M} = 0$. Thus, some dimensionless components of initial vector $c^{(0)}$ are unknown: the forces $\overline{F}_x$, $\overline{F}_y$ and coordinate $\bar{x}_K = \frac{x_K}{D}$. Boundary conditions at point A of the coordinate $\bar{s} = \frac{l - x_K}{D}$ are the same as in Stage I. The stage ends when the sliding part of the elastica contacts the base at the point S [Fig. 4(b)]. Figure 4(b) includes some parameters, which characterize the fold created i.e. the width x_F and height y_F. According to calculations, $x_F = 1.185$ m and $y_F = 0.293$ m. The corresponding length of elastica (l_{g2}) is called the border length of Stage II. In this example, l_{g2} = 3.416 m.

Numerical solutions of differential Eq. (6) give the dimensionless forces $\overline{F}_x$, $\overline{F}_y$ within the points A and B and the bending moment $\overline{M}$ at any point of elastica. To receive the values in SI units, let us multiply the equations by the coefficients according to Eq. (5). Let the linear weight density be q = 0,01 N/m. The results for two exemplary lengths are listed in Table 3.

The shooting method is divergent for some small values of D, order of less than 0.122 m, and the decreased bending rigidity C. Let us explain this problem by means of sensitivity analysis.

3.2 Sensitivity Analysis of Elastica Equations

Introducing the shooting method to analyze the Stage I [Fig. 3(b)], we have to determine the control parameters i.e. the dimensionless forces $\overline{F}_x$, $\overline{F}_y$ and the angle φ at one end of the elastica. Simultaneously, at the other end the boundary conditions [Eq. (7)] should be satisfied. The equations can be solved with a predetermined accuracy if the control parameters are recorded with an accuracy better than the predisposed number of significant digits in a digital computer. On the contrary, the solution of equations with the assumed accuracy is impossible. A small change of the control parameters causes the large change of the solutions and the calculation algorithm is unstable. All variables in Eq. (6), such as $\overline{M}$, $\overline{F}_x$, $\overline{F}_y$, φ, $\bar{x}$ and $\bar{y}$ depend on three control parameters $\overline{F}_x^0$, $\overline{F}_y^0$ and φ^0 and coordinate s. Let us determine the bending parameters of elastica equations (length - stiffness) in respect of the algorithm unstability.

The sensitivity is analyzed using the direct approach. Let us determine the variational form of differential Eq. (6) in respect of the particular parameter defined generally by v. The corresponding variation of bending moment $\frac{d\overline{M}}{d\bar{s}}$ has the following form:

Table 3 — Boundary conditions at the end points of elastica during Stage I

Length	Point A	Point B
l = 1.01 m	F_x = -0.00069 N F_y = 0.00175 N M = -0.00035 Nm	F_x = -0.00069 N F_y = 0.01185 N
l = lg1	F_x = -0.00378 N F_y = -0.00437 N M = -0.00126 Nm	F_x = -0.00378 N F_y = 0.00899 N

$$\frac{d}{d\bar{s}}\left(\frac{\partial \bar{M}}{\partial v}\right)=\frac{\partial \bar{F}_x}{\partial v}\sin\varphi+\bar{F}_x\frac{\partial\varphi}{\partial v}\cos\varphi+\frac{\partial \bar{F}_y}{\partial v}\cos\varphi-\bar{F}_y\frac{\partial\varphi}{\partial v}\sin\varphi\,. \quad \ldots (8)$$

The notations concerning the partial derivatives of parameters with respect to v have the following form:

$$\frac{\partial\varphi}{\partial v}=\tilde{\varphi},\quad \frac{\partial \bar{M}}{\partial v}=\tilde{\bar{M}},\quad \frac{\partial \bar{F}_x}{\partial v}=\tilde{\bar{F}}_x,$$
$$\frac{\partial \bar{F}_y}{\partial v}=\tilde{\bar{F}}_y,\quad \frac{\partial \bar{x}}{\partial v}=\tilde{\bar{x}},\quad \frac{\partial \bar{y}}{\partial v}=\tilde{\bar{y}}\,. \quad \ldots (9)$$

The result are six differential equations of the unknowns derivatives [Eq. (9)], as shown below:

$$\frac{d\tilde{\varphi}}{d\bar{s}}=\tilde{\bar{M}},\quad \frac{d\tilde{\bar{M}}}{d\bar{s}}=\tilde{\bar{F}}_x\cdot\sin\varphi+\tilde{\varphi}\cdot\bar{F}_x\cos\varphi+\tilde{\bar{F}}_y\cdot\cos\varphi-\tilde{\varphi}\cdot\bar{F}_y\sin\varphi,$$
$$\frac{d\tilde{\bar{F}}_x}{d\bar{s}}=0,\quad \frac{d\tilde{\bar{F}}_y}{d\bar{s}}=0,\quad \frac{d\tilde{\bar{x}}}{d\bar{s}}=-\tilde{\varphi}\cdot\sin\varphi,\quad \frac{d\tilde{\bar{y}}}{d\bar{s}}=\tilde{\varphi}\cdot\cos\varphi\,. \quad \ldots (10)$$

Next, the same variations are determined for the boundary conditions at the point B ($\bar{s}=0$) listed in Table 4.

To analyze the sensitivity, it is necessary to solve a system of Eq. (6) with a set of boundary conditions defined in Table 4 for each control parameter v. The values of forces $\bar{F}_x$, $\bar{F}_y$ and angle φ are determined by means of solution of these equations.

Table 4 — Variations in initial conditions for three parameters (v)

Parameters	$\tilde{\varphi}^0$	$\tilde{\bar{F}}_x^0$	$\tilde{\bar{F}}_y^0$	$\tilde{\bar{M}}^0$	$\tilde{\bar{x}}^0$	$\tilde{\bar{y}}^0$
$v=\varphi^0$	1	0	0	0	0	0
$v=\bar{F}_x^0$	0	1	0	0	0	0
$v=\bar{F}_y^0$	0	0	1	0	0	0

The obtained derivatives at an optional elastica point $\tilde{\varphi}$, $\tilde{\bar{M}}$, $\tilde{\bar{F}}_x$, $\tilde{\bar{F}}_y$, $\tilde{\bar{x}}$ and $\tilde{\bar{y}}$ describe the changes in the angle (φ), bending moment ($\bar{M}$), forces ($\bar{F}_x$, $\bar{F}_y$) and coordinates ($\bar{x}$, $\bar{y}$) in respect of an infinitesimal change of the corresponding control parameter v at point B ($\bar{s}=0$). The most important is to specify the changes at the second end of elastica (point A), to satisfy the appropriate boundary conditions [Fig. 3(b)].

To examine the sensitivity of differential equations with increasingly smaller values of parameter D, the Eq. (6) is solved with the appropriate initial conditions using the fourth-order Runge-Kutta method. The assumed data are as follows: the length l = 1.01 m, the bending length 0.122 m ≤ D ≤ 0.400 m. The variations of elastica derivatives at the second end (point A) versus the bending length D are depicted in Fig. 5(a)-(c).

The sensitivity of coordinates $\bar{x}$, $\bar{y}$ and angle φ to changes of initial values $\bar{F}_x^0$, $\bar{F}_y^0$, φ^0 increases rapidly close to bending length D = 0.16 m. For the value D = 0.12 m, the system is more and more sensitive. Therefore, even minimal change of initial values $\bar{F}_x^0$, $\bar{F}_y^0$, φ^0 in shooting method causes a very large change in the values $\bar{x}$, $\bar{y}$ and φ in point A. Thus, the shooting method is ineffective at small values of D, particularly near D = 0.12 m. The values below this limit cause the iterative process to be divergent, and therefore, other computational methods should be applied.

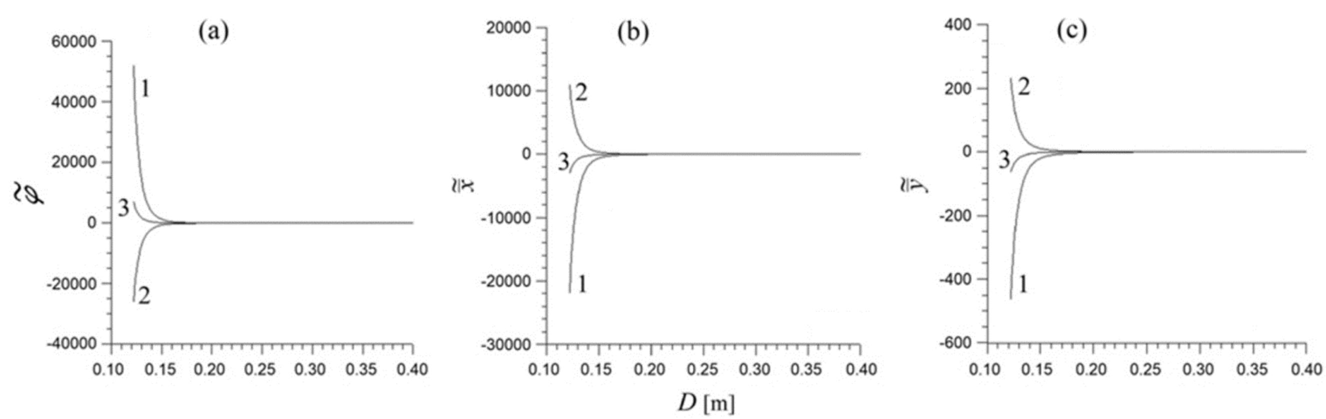

Fig. 5 — Sensitivity in respect of changes of initial values (1) $\bar{F}_x^0$, (2) φ^0, (3) $\bar{F}_y^0$: (a) of angle φ, (b) of coordinate $\bar{x}$, and (c) of coordinate $\bar{y}$

3.3 Proposal of New Test Concerning Bending Rigidity

Results of numerical experiments prove that the width of fold is directly proportional to the bending rigidity. Based on observations of similar real phenomena, it is expected that an increase in stiffness causes an increase in the width of formed fold. Therefore, the width may be considered as a parameter characterizing the bending rigidity, similarly as introduced by Peirce bending length *D*. The height (*H*) of fabric feeding also affects the width of the fold.

The problem has been solved numerically for different values of height (*H*), which confirms the assumption. The changes of the coordinate describing the forming fold x_F are shown in Fig. 6(a), considering height *H* for several bending lengths *D*.

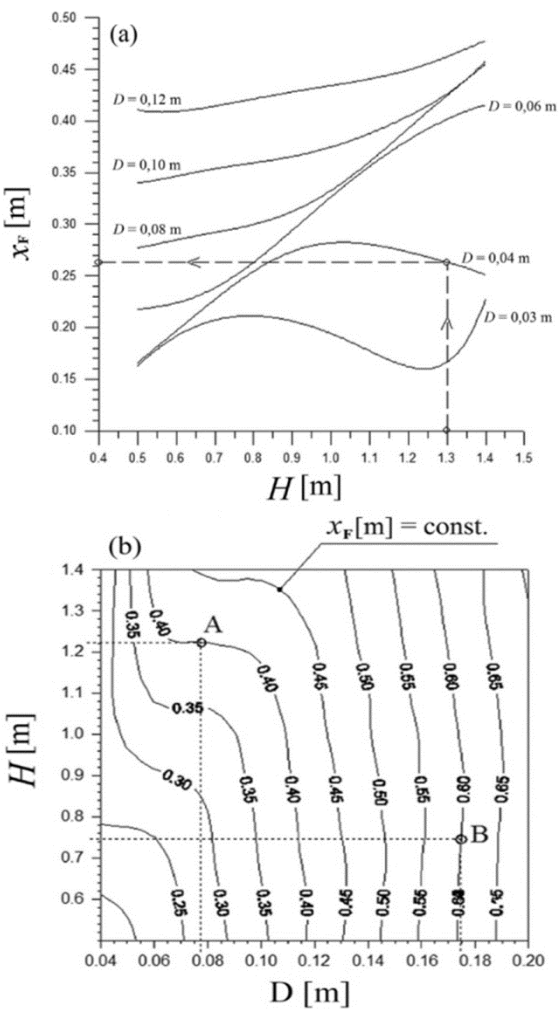

Fig. 6 — (a) Width of forming fold x_F versus height *H* of fabric feeding for different bending length (*D*), and (b) contour-lines plan of width (x_F) as a function of two variables *H* and *D*

Figure 6(b) illustrates the calculation results in contour-lines plan, that is the contours of width coordinate of forming fold (x_F) versus two variables *H* and *D*.

In the neighbourhood of the specified point A at a fixed height (*H*), the different bending lengths (*D*) generate nearly the same width of the fold x_F [Fig. 6(b)]. The reason is almost horizontal course of the contour. However, in the neighborhood of the point B, the width of fold corresponds unequivocally to the bending length *D*. The larger the width of the fold, the greater is the bending length. Condition illustrated by the point A is not convenient, whereas point B secures the advantageous assessment of bending stiffness based on the width of forming fold.

Therefore, it is convenient to evaluate the bending stiffness based on the measured width of the forming fold x_F. To apply the method correctly, the height (*H*) should be defined to ensure the unambiguous results for a wide range of bending length (*D*). According to Fig. 6(b), the optimal height is of the order of 0.5 m, and all measuring points are similar to point B. The width of the fold determined for *H* = 0.5 m is found (Fig. 7) as almost linear.

Peirce introduced the bending length as a representative measure of the elastica stiffness. The current proposal determines the width of fold x_F as a measure of bending stiffness *C*. The sample can be

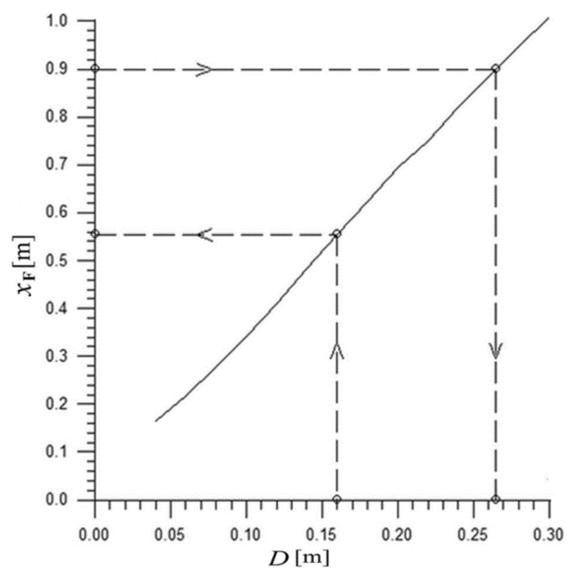

Fig. 7 — Width of fold x_F as a function of bending length *D* for height of fabric feeding *H* = 0.5 m

subjected to bending in both directions (right and left) in a wide range of curvatures. This is an interesting alternative to Peirce's test, which introduced the test specimen subjected to a slight bending in one direction only.

4 Conclusion

The ambiguos solutions of the nonlinear differential equations describing the elastica equilibrium affect essentially the convergence of numerical algorithm. The shooting method is divergent for the infinitesimal values of bending stiffness. The sufficient convergence is assured only by the finite difference method.

The shooting method is effective for a specific range of stiffness which can be estimated using the sensitivity analysis of the solutions for the bending rigidity. Numerical tests have proved that this method is beneficial in respect of computation time and computer memory usage. The finite difference approach is slower due to the large number of unknowns and requires a considerable computer memory. However, the finite difference method is convergent for the smaller bending stiffnesses. The problem of free folding was solved numerically. It is an interesting proposal to evaluate the bending stiffness based on the measured width of the fold. The optimal width was proposed ($x_F = 0.5$ m) for a range of bending stiffnesses appearing in textile industry. The sensitivity was analyzed to determine the application range for the bending test. The sensitivity approach can be additionally applied to optimize the shape of elastica for the assumed material characteristics which can be an interesting extension of current work.

The initial objective of textile mathematical modelling is always to simplify the real textile structure and deformation and to create an adequate model. The examples of elastica bending show that there are great opportunities to model and simulate its behaviour under different conditions.

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