Indian Journal of Pure & Applied Physics
Vol. 56, January 2018, pp. 63-68

Application of homotopy perturbation method for MHD free convection of water at 4 ºC through porous medium bounded by a moving vertical plate

A K Jhankal
Army Cadet College, Indian Military Academy, Dehradun 248 007, India

Received 20 January 2017; accepted 14 December 2017

An analysis has been made to study the problem of two-dimensional magnetohydrodynamic (MHD) free convection of water at 4 ºC through porous medium bounded by a moving vertical plate. The governing partial differential equations have been transformed into self-similar ordinary differential equations using similarity transformations before being solved by He's homotopy perturbation method (HPM). The main advantage of HPM is that it does not require the small parameters in the equations and hence the limitations of traditional perturbation can be eliminated. The results reveal that the proposed method is very effective and simple and can be applied to other nonlinear problems. A parametric study of all involved physical parameters has been conducted and a representative set of numerical results for the velocity, temperature and skin-friction has been illustrated graphically. Physical aspects of the problem have also been discussed.

Keywords: MHD, HPM, Porous medium, Moving vertical plate

1 Introduction

The subject of MHD is largely perceived to have been initiated by Swedish electrical engineer Hannes Alfveén[1] in 1942. Under the influence of magnetic field, electrically conducting fluid induces currents. This also creates force on the fluid. MHD can be used for the control of fluid flow. MHD has been employed for many engineering applications in aerodynamic heating, electrostatic precipitation, petroleum industry, purification of oil and fluid droplets and sprays, etc. The confinement of hot plasma is of great importance in nuclear fusion devices where vast amount of energy is released. The MHD may be used for magnetically pinching the hot plasma[2]. In addition, the MHD flows of electrically conducting fluid through porous media has been motivated by its immense importance and continuing interest in many engineering and technological field, for example, soil mechanics, petroleum engineering, transpiration cooling, food preservation, cosmetic industry blood flow and artificial dialysis etc.

The theory of laminar flows through, homogeneous media is based on an experiment originally conducted by Darcy[3]. Most of the past studies on porous media are concerned with the case of fluid having a linear relationship between density and temperature. Nilsen and Storesletten[4], Ni and Beckermann[5], Nield and Bejan[6], Trew and Mckibbin[7], Ingham and Pop[8]. Goren[9] have shown that usual Navier-Stokes equations are not applicable for studying the flow of water at 4 ºC where the density variation is given by:

$$\Delta\rho = -\rho\gamma(\Delta T)^2 \text{ , with } \gamma = 8 \times 10^{-6} (°\mathrm{C})^{-2}$$

Thus convection in porous medium saturated with water near 4 ºC, point at which the density of water reaches a maximum value, behaves in a complicated manner. Convection in porous media saturated with cold water and confined in rectangular cavities has been investigated by Lin and Nansteel[10], Robillard and Vasseur[11], Tong and Koster[12]. Transient natural convection of water near its density extremism in a rectangular cavity filled with an isotropic porous medium was investigated numerically by Chang and Yang[13], Zheng *et al.*[14] has studied the steady free convection flow of water near 4 °C on vertical and horizontal plates when the temperature of the plate is varying as a power of the distance along the plate from the leading edge. Pop and Raptis[15] investigated the transient free convection of water near 4 °C over a doubly infinite vertical porous plate. The combined convection flow of water near 4 °C through a porous medium bounded by a vertical plate was studied by Raptis and Pop[16]. Raptis and Perdikis[17] also studied the free convection flow of water near 4 °C past an infinite porous plate with constant suction and free stream-

E-mail: anujjhankal@yahoo.com

velocity. Singh and Raptis[18] studied the free convection flow of water near 4 °C past an infinite vertical porous plate with constant heat flux. Ling *et al.*[19] studied the steady mixed convective water flow over a vertical plate in a porous medium near 4°C when the wall temperature and surface heat flux varies. The natural convection in a triangular enclosure filled with porous media saturated with water near 4°C was made by Oztop *et al.*[20]. Khan and Gorla[21] studied the mixed convection of water near 4 °C along a wedge with variable surface temperature in porous medium and the non similar solution for mixed convection of water near 4 °C in a porous medium. Furthermore, some important contributions in the MHD free convective flow of water near 4 °C through porous medium under various physical situations were made by Jat and Jhankal[22] and Guedda *et al.*[23].

Most of the problems in MHD area are nonlinear. Except a limited number of these problems have precise analytical solution, most of them do not have exact solution, and so these nonlinear equations should be solved using other proper methods. Most of the scientists believe that the combination of numerical and semi-exact analytical methods can lead to applicable results. In this paper one of the semi-exact method which is called He's homotopy perturbation method (HPM) has been introduced and its application for the problem of the two-dimensional steady free convection of water at 4 ºC through highly porous medium bounded by a moving vertical plate in presence of transverse magnetic field is presented and discussed. The initial work in HPM was studied by He[24-26] and after that these investigations inspired a lot of researchers Beléndez *et al.*[27], Ganji and Ganji[28], Jhankal[29-31] and many others to solve nonlinear equations with this method.

To illustrate the basic ideas of the HPM, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0, r \in \Omega \quad \dots (1)$$

Subject to the boundary conditions:

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0, r \in \Gamma \quad \dots (2)$$

where A is a general differential operator, B is a boundary operator, *f(r)* is a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts which are L and N, where L is linear and N is nonlinear. Therefore Eq. (1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, r \in \Omega \quad \dots (3)$$

By the homotopy perturbation technique, we construct a homotopy:

$v(r,p): \Omega \times [0,1] \to R$, which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, p \in [0,1], r \in \Omega \quad \dots (4)$$

where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation that satisfies the boundary condition. Obviously, from these definitions we will have:

$$H(v, 0) = L(v) - L(u_0) = 0$$

$$H(v, 1) = A(v) - f(r) = 0$$

The changing process of p from zero to one is just that of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopy. According to the HPM, we can first use the embedding parameter p as a "small parameter" and assuming that the solution of Eq. (4) can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 \dots \quad \dots (5)$$

Setting $p = 1$, results in the approximate solution of Eq. (1):

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \quad \dots (6)$$

The convergence and stability of this method was shown in Hosein *et al.*[32].

2 Mathematical Formulation of the Problem

Consider a two-dimensional free convection flow of water at 4 °C temperature at which the density reaches its maximum value. The flow is through a highly porous medium so that a force $\mu\bar{u}/\bar{K}$ is developed at each particle. The bounded plate is moving in its own plane the plane with a constant velocity U_0 and is subjected to a constant normal suction. The axis of $\bar{x}$ be taken along the plane of the wall and $\bar{y}$ axis perpendicular to it, the applied magnetic field is of uniform strength and is applied transversely to the $\bar{x}$ direction of the flow. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy (Pai[33], Schlichting[34], Bansal[35]) under the influence of externally imposed transverse magnetic field (Jeffery[36], Bansal[37]) are:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad \dots (7)$$

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\gamma(\bar{T}-\bar{T}_\infty)^2 + \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \nu\frac{\bar{u}}{\bar{K}} - \frac{\sigma_e B_0^2 \bar{u}}{\rho} \quad \dots (8)$$

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \quad \dots (9)$$

Along with the appropriate boundary conditions for the problem are given by:

$$y = 0: \bar{u} = U_0, \bar{T} = \bar{T}_w$$

$$y \to \infty: \bar{u} \to 0, \bar{T} \to \bar{T}_\infty \quad \dots (10)$$

The last two terms on the right hand side of Eq. (8) signify the additional resistance due to the porous medium, with permeability $\bar{K}$, and the electromagnetic body force which acts on the fluid elements, respectively. Also, the Joule heating term in the energy Eq. (9) is assumed to be negligible.

3 Analysis

From the continuity Eq. (7), for constant suction, we get $v = -v_0$. The momentum and energy equations can be transformed into the corresponding ordinary differential equation by using the following non-dimensional parameters:

$$y = U_0\bar{y}/\nu,\ u = \bar{u}/U_0,\ \theta = (\bar{T}-\bar{T}_\infty)/(\bar{T}_w - \bar{T}_\infty) \quad \dots (11)$$

The transformed ordinary differential equations are:

$$u'' + \lambda u' = -Gr\theta^2 + \frac{u}{K} + Mu \quad \dots (12)$$

$$\theta'' + Pr\lambda\theta' = -PrEc(u')^2 \quad \dots (13)$$

The transformed boundary conditions are:

$$u(0) = 1, \theta(0) = 1 \text{ and}$$

$$u(\infty) \to 0, \theta(\infty) \to 0. \quad \dots (14)$$

where prime denotes differentiation with respect to y, $M = \sigma_e B_0^2 \upsilon/\rho U_0^2$ is the dimensionless magnetic parameter, $Pr = \mu c_p/\kappa$ is the Prandtl number, $\lambda = v_0/U_0$ is the Suction parameter, $K = \bar{K}U_0^2/v^2$ is the permeability parameter, $Ec = U_0^2/C_p(\bar{T}_w - \bar{T}_\infty)$ is the Eckert number, and $Gr = vg\gamma(\bar{T}_w - \bar{T}_\infty)^2/U_0^2$ is the Grashof number.

According to the HPM, the homotopy form of Eqs (12) and (13) are constructed as follows:

$$(1-p)\left(u'' + \lambda u' - \frac{1}{K}u - Mu\right) + p(u'' + \lambda u' - \frac{1}{K}u - Mu + Gr\theta^2) = 0 \quad \dots (15)$$

$$(1-p)[\theta'' + Pr\lambda\theta'] + p\left[\theta'' + Pr\lambda\theta' + PrEcu'^2\right] = 0 \quad \dots (16)$$

We consider f and θ as the following:

$$u = u_0 + pu_1 + p^2u_2 \dots$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \dots \quad \dots (17)$$

By substituting Eq. (17) into (15) and (16), and then:
(I) Terms independent of p give:

$$u_0'' - \lambda u_0' - \frac{1}{K}u_0 - Mu_0 = 0 \quad \dots (18)$$

$$\theta_0'' + Pr\lambda\theta_0' = 0 \quad \dots (19)$$

The boundary conditions are:

$$u_0(0) = 1, u_0(\infty) = 0, \theta_0(0) = 1, \theta_0(\infty) = 0 \quad \dots (20)$$

(II) Terms containing only p give:

$$u_1'' - \lambda u_1' - \frac{1}{K}u_1 - Mu_1 + Gr\theta_0^2 = 0 \quad \dots (21)$$

$$\theta_1'' + Pr\lambda\theta_1' + PrEcu'^2 = 0 \quad \dots (22)$$

The boundary conditions are:

$$u_1(0) = 0, u_1(\infty) = 0, \theta_1(0) = 0, \theta_1(\infty) = 0 \quad \dots (23)$$

Solving Eqs (18)-(19) and (21)-(22) with boundary conditions Eqs (20) and (23), respectively, we have:

$$u_0 = C_1e^{n_1 y} + C_2e^{n_2 y} \quad \dots (24)$$

$$u_1 = C_5e^{n_1 y} + C_6e^{n_2 y} + \frac{GrC_3^2}{\frac{1}{K}+M} - \frac{GrC_4^2e^{-2\mathrm{Pr}\lambda}}{4Pr^2\lambda^2 - 2Pr\lambda^2 - \left(\frac{1}{K}+M\right)} - \frac{2GrC_3C_4e^{-\mathrm{Pr}\lambda y}}{Pr^2\lambda^2 - Pr\lambda^2 - \left(\frac{1}{K}+M\right)} \quad \dots (25)$$

$$\theta_0 = C_3 + C_4e^{-\mathrm{Pr}\lambda y} \quad \dots (26)$$

$$\theta_1 = C_7 + C_8 e^{-\Pr\lambda y} - \frac{PrEcC_1^2 n_1^2 e^{2n_1 y}}{4n_1^2 + 2Pr\lambda n_1} - \frac{PrEc\ {}_2^2 n_2^2 e^{2n_2 y}}{4n_2^2 + 2Pr\lambda\ {}_2} - \frac{2PrEcC_1 C_2 n_1 n_2 e^{(n_1+n_2)y}}{(n_1+n_2)^2 + Pr\lambda(n_1+n_2\}} \quad \ldots (27)$$

The constant coefficients, can be calculated using boundary conditions, the boundary condition η=∞ were replaced by those at η=5 in accordance with standard practice in the boundary layer analysis. If *p*→1, we find the approximate solution of Eqs (15) and (16).

The constant coefficients $A_i (i = 1, 2, 3, 4)$, C_j $(j = 1, 2, 3, \ldots, 8)$, n_1 and n_2 are defined as in the appendix.

4 Results and Discussion

In order to get a physical insight into the problem, numerical computations are performed for various values of the physical parameters involved in the equations viz. magnetic parameter *M*, suction parameter λ, permeability parameter *K* and fixed values of Grashof number (*Gr*) and Eckert number (*Ec*). Since the value of Prandtl number (*Pr*) for water at 4 °C is 11.4, the numerical results are obtained for *Pr*=11.4. Calculated results are presented in the Figs 1 to 4 to understand the effect of parameters on velocity and temperature field.

The impact of magnetic parameter *M* on the velocity profile is very significant in practical point of view. In Figs 1 and 2, the variation in velocity field for several values of *M* is presented. It is observed that the dimensionless velocity decreases with increasing values of *M*, this happens due to Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid (water). This force acts on the fluid (water) creating a drag-like effect that slows down the motion of the fluid in the boundary layer. In addition, the reversed phenomenon is observed for increasing values of *K* while λ is fixed. This is because with a rise in permeability of the medium, the regime becomes more porous. As a consequence, the Darcian body force decreases in magnitude (as it is inversely proportional to the permeability). The Darcian resistance acts to decelerate the fluid particles in continua. This resistance diminishes as permeability of the medium increases. So progressively less drag is experienced by the flow and flow retardation is thereby decreased. Hence the velocity of the fluid (water) increases as the permeability parameter increases.

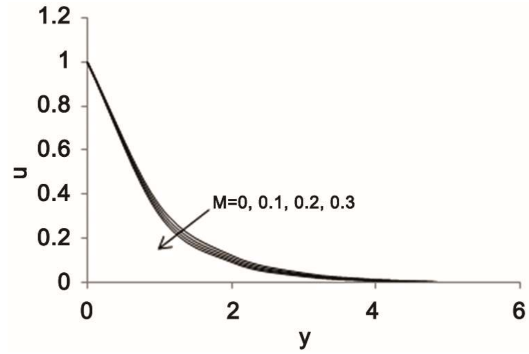

Fig. 1 — Effect of *M* on the velocity profile for λ=0.1 and *K*=1.

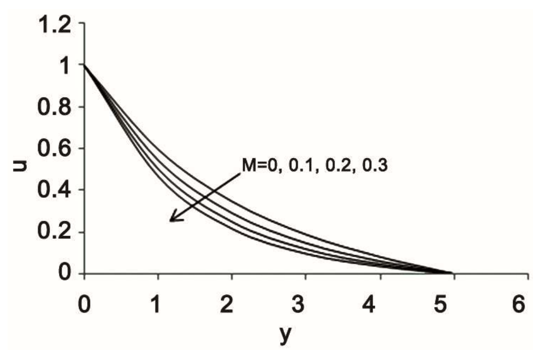

Fig. 2 — Effect of *M* on the velocity profile for λ=0.1 and *K*=5.

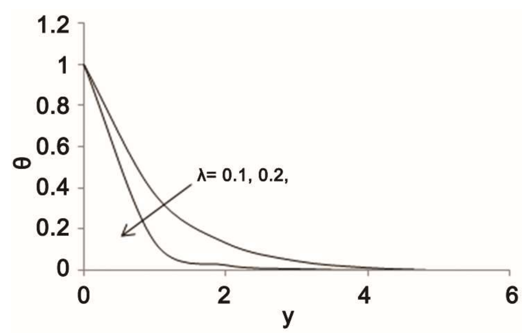

Fig. 3 — Effect of λ on the temperature profile for *K*= 1, *M*=0.2, *Gr*=5, *Ec*=0.05 and *Pr*=11.4.

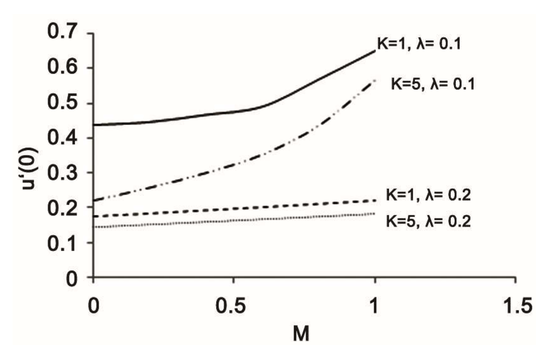

Fig. 4 — Skin friction against *M* for different values of *K* and λ.

Figure 3 which illustrates the effect of suction parameter λ on the temperature profiles. We infer from this figure that temperature decreases with an increase in the suction parameter. From this plot it is evident that large values of suction parameter result in thinning of the thermal boundary layer. The explanation for such behaviour is that the fluid is brought closer to the surface and reduces the thermal boundary layer thickness.

The local skin friction coefficient against magnetic parameter *M* for various values of suction parameter λ and permeability parameter *K*, is illustrated in Fig. 4. It is noted that the skin friction coefficient decreases with an increase in λ. Similar results occur for increasing values of *K*.

5 Conclusions

A mathematical model has been presented for the MHD free convection of water at 4°C through porous medium bounded by a moving vertical plate. The self-similar equations are obtained using similarity transformations. The nonlinear ordinary differential equations were solved by homotopy perturbation method and the influence of several physical parameters controlling the velocity and temperature profiles are shown graphically and discussed briefly. As the magnetic parameter increases, we can find the velocity profile decreases in the flow region. Thus we conclude that we can control the velocity field by introducing magnetic field, but reversed phenomenon is observed for increasing value of permeability parameter. The temperature decreases with an increase in the suction parameter. It is also noted that, the skin friction coefficient decreases with an increase in suction parameter. Similar results occur for increasing values of permeability parameter.

The homotopy perturbation method (HPM) suggested in this paper is an efficient method for obtaining the solution of nonlinear differential equations. Therefore, this method is a powerful mathematical tool to solve any system of linear and nonlinear differential equations.

Nomenclature

B_0 Constant applied magnetic field, [Wb m^{-2}]
C_p Specific heat at constant pressure, [J kg^{-1}K^{-1}]
Ec Eckert number ($= U_\infty^2/C_p(T - T_\infty)$), [−]
g Gravity acceleration, [m s^{-2}]
Gr Grashof number ($= \upsilon g\gamma(\bar{T}_w - \bar{T}_\infty)^2/U_0^2$), [−]
K Permeability parameter ($= \bar{K}U_0^2/v^2$), [−]
M Magnetic parameter ($= \sigma_e B_0^2 \upsilon/\rho U_0^2$), [−]
Pr Prandtl number ($= \mu C_p/\kappa$), [−]
T Temperature of the fluid, [K]
u, v Velocity component of the fluid along the *x* and *y* directions, respectively, [m s^{-1}]
U_0 Characteristic velocity, [m s^{-1}]
x, y Cartesian coordinates along the surface and normal to it, respectively, [m]

Greek symbols

λ Suction parameter, [$= v_0/U_0$]
ρ Density of the fluid, [kg m^{-3}]
γ Thermal expansion coefficient, [K^{-1}]
μ Viscosity of the fluid, [kg m s^{-1}]
σ_e Electrical conductivity, [m^2 s^{-1}]
κ Thermal conductivity, [W m^{-2}K^{-4}]
υ Kinematic viscosity, [m^{2}s^{-1}]
θ Dimensionless temperature, [$= (\bar{T} - \bar{T}_\infty)/(\bar{T}_w - \bar{T}_\infty)$]

Superscript

$'$ Derivative with respect to y
− Dimensional quantities

Subscripts

w Properties at the plate
∞ Free stream condition

Appendix

$$A_1 = \frac{GrC_3^2}{\left(\frac{1}{K}+M\right)} - \frac{GrC_4^2}{4Pr^2\lambda^2-2Pr\lambda^2-\left(\frac{1}{K}+M\right)} - \frac{2GrC_3C_4}{Pr^2\lambda^2-Pr\lambda^2-\left(\frac{1}{K}+M\right)},$$

$$A_2 = \frac{GrC_3^2}{\left(\frac{1}{K}+M\right)} - \frac{GrC_4^2e^{-10\mathrm{Pr}}}{4Pr^2\lambda^2-2Pr\lambda^2-\left(\frac{1}{K}+M\right)} - \frac{2GrC_3C_4e^{-5\mathrm{Pr}\lambda}}{Pr^2\lambda^2-Pr\lambda^2-\left(\frac{1}{K}+M\right)},$$

$$A_3 = \frac{PrEcC_1^2n_1^2}{4n_1^2+2Pr\lambda n_1} + \frac{PrEcC_2^2n_2^2}{4n_2^2+2Pr\lambda n_2} + \frac{2PrEcC_1C_2n_1n_2}{(n_1+n_2)^2+Pr\lambda(n_1+n_2)},$$

$$A_4 = \frac{PrEcC_1^2n_1^2e^{10n_1}}{4n_1^2+2Pr\lambda n_1} + \frac{PrEcC_2^2n_2^2e^{10n_2}}{4n_2^2+2Pr\lambda n_2} + \frac{2PrEcC_1C_2n_1n_2e^{5(n_1+n_2)}}{(n_1+n_2)^2+Pr\lambda(n_1+n_2)},$$

$$C_1 = \frac{e^{5n_2}}{e^{5n_2}-e^{5n_1}}, \quad C_2 = 1 - C_1, \quad C_3 = \frac{e^{-5\mathrm{Pr}\lambda}}{e^{-5\mathrm{Pr}\lambda}-1}, \quad C_4 = 1 - C_3,$$

$$C_5 = \frac{A_2 - A_1e^{5n_2}}{e^{5n_2}-e^{5n_1}}, \; C_6 = -C_5 - A_1, \; C_7 = \frac{A_3e^{-5\mathrm{Pr}\lambda} - A_4}{e^{-5\mathrm{Pr}\lambda}-1},$$

$C_8 = A_3 - C_7,$

$$n_1 = \frac{-\lambda + \sqrt{\lambda^2 + 4\left(\frac{1}{K} + M\right)}}{2},\ n_2 = \frac{-\lambda - \sqrt{\lambda^2 + 4\left(\frac{1}{K} + M\right)}}{2}.$$

References

1 Alfven H, *Nature*, 150 (1942) 405.
2 Chandrasekhar S, *Hydrodynamic and hydromagnetic stability*, (Oxford University Press: England), 1961.
3 Darcy H P G, *Les fountains publiques dela ville de Dijam Dalmont*, Pariser, 1856.
4 Nilsen T & Storesletten L, *ASME J Heat Transfer*, 112 (1990) 396.
5 Ni J & Beckermann C, *ASME J Heat Transfer*, 113 (1991), 1033.
6 Nield D A & Bejan A, *Convection in porous media*, (Springer: N Y), 1992.
7 Trew M & McKibbin R, *Transport Porous Media,* 17 (1994) 271.
8 Ingham D B & Pop I, *Transport phenomena in porous media*, (Pergamon: Oxford), 1998.
9 Goren S, *Chem Eng Sci*, 21 (1966) 515.
10 Lin D S & Nansteel N W, *Int J Heat Mass Transfer*, 30 (1987) 2319.
11 Robillard L & Vasseur P, *Freezing and melting heat transfer in engineering,* (Hemisphere: Washington D C), 1991.
12 Tong W & Koster J, *Int J Heat Mass Transfer*, 37 (1994) 927.
13 Chang W J & Yang D F, *Heat Transfer,* 28 A (1995) 619.
14 Zheng W, Robillard L & Vasseur P, *Int J Heat Mass Transfer,* 44 (2001) 3463.
15 Pop I & Raptis A, *J Heat Transfer Trans ASME*, 104 (1982) 800.
16 Raptis A & Pop I, *Lett Heat Mass Transfer*, 9 (1982) 309.
17 Raptis A & Perdikis C, *Bulletin de la Classe des Sciences Academie Royale de Belgique*, 5th Series-Tome LXVIII (1982-1984) 259.
18 Singh A & Raptis A, *Astrophys Space Sci*, 104 (1984) 399.
19 Ling S, Nazar R & Pop I, *Transport Porous Media*, 69 (2007) 359.
20 Oztop H, Varol Y & Pop I, *Energy Convers Manag*, 50 (2009) 1473.
21 Khan W & Gorla R, *Inte J Therm Phys*, 32 (2011) 2079.
22 Jat R N & Jhankal A K, *Ganita Sandesh*, 17 (2003) 61.
23 Guedda M, Aly E & Quahsine A, *Appl Math Model*, 35 (2011) 5182.
24 He J H, *Comput Math Appl*, 57 (2009) 410.
25 He J H, *J Comput Math App Mech Eng*, 167 (1998) 57.
26 He J H, *Int J Nonlinear Sci Numer Simul*, 1 (2000) 51.
27 Beléndez A, Beléndez T, Márquez A & Neipp C, *Chaos Solitons Fractals*, 37 (2008) 770.
28 Ganji Z Z & Ganji D D, *Int J Nonlinear Sci Numer Simul*, 9 (2008) 415.
29 Jhankal A K, *J Appl Fluid Mech*, 7 (2014) 177.
30 Jhankal A K, *J Appl Fluid Mech*, 8 (2015) 571.
31 Jhankal A K, *Chem Eng Res Bull*, 18 (2015) 12.
32 Hosein N S H, Ranjbar A N, Ganji D D, Soltani H & Ghasemi J, *Phys Lett A*, 372 (2008) 2855.
33 Pai S I, *Viscous flow theory: i – laminar flow* (D Van Nostrand Co: New York), 1956.
34 Schichting H, *Boundary layer theory,* 6th Edn, (McGraw-Hill: New York), 1964.
35 Bansal J L, *Viscous fluid dynamics*, (Oxford & IBH Pub Co: New Delhi) 1977.
36 Jeffery A, *Magnetohydrodynamics*, (Oliver and Boyed: New York), 1966.
37 Bansal J L, *Magnetofluiddynamics of viscous fluids*, (Jaipur Publishing House: Jaipur), 1994.