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Amplitude modulation and demodulation of a coherent electromagnetic wave in magnetized doped III-V semiconductors

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This present paper is concerned with the analytical study of the amplitude modulation and demodulation of a coherent electromagnetic wave in magnetized doped III-V semiconductors. Utilizing the hydrodynamic model of a semiconductor plasma, the modulation indices for upper (+) and lower (–) side bands have been obtained. The incorporation of carrier diffusion in the nonlinear laser-semiconductor interaction adds new dimension to the analysis. The numerical estimations have been made for n-InSb crystal at liquid nitrogen temperature illuminated by frequency doubled pulsed 10.6 µm CO_2 laser. The problem has been analyzed in two different wave regimes (in the presence as well as absence of phenomenological acoustic damping parameter (Γ_a) over a wide range of externally applied magnetic field (incorporated in terms of cyclotron frequency ω_c). The results indicate that in absence of damping parameter, the absorption of coherent electromagnetic radiation takes place completely in all possible wavelength regimes when $\omega_c \sim (\nu^2 + \omega_0^2)^{1/2}$; ν and ω_0 being the electron collision frequency and pump frequency, respectively. Moreover, the carrier diffusion modifies amplitude modulation and demodulation processes significantly. The damping parameter additionally assumes a significant role in choosing the range of parameters and selecting the modulated side band mode.

Keywords: Amplitude modulation/demodulation, Diffusion, Acousto-electric effects, III-V semiconductors

1 Introduction

Laser-matter interaction has been playing an important role in diverse areas of scientific research owing to its numerous applications in processing of materials and fabrication of optoelectronic devices[1,2]. Nonlinear crystals provide a compact and less expansive medium to demonstrate nonlinear optical phenomena. In a nonlinear crystal, the breakdown of superposition principle leads to the interaction among waves of different frequencies. There exist a variety of nonlinear interactions which may be classified as modulation interactions. The subsequent amplification of decay channels by modulation interactions are generally known as an instability of wave propagating in nonlinear dispersive medium such that the steady state becomes unstable and develops into a temporally modulated state[3]. The idea of modulation instability emerges from a space-time analogy that exists when the dispersion is supplanted by diffraction[4]. An outstanding case of modulation instability is the instability of a plane wave in a self-focusing Kerr-medium[5].

Modulation instability of propagating waves has been a subject of profound enthusiasm since the origin of nonlinear optics with exceptional accentuation to diffraction and wave guiding processes. This is because of the way that the scattering of light from sound or low frequency electromagnetic wave bears an advantageous method to control the frequency, intensity and direction of propagation of an coherent electromagnetic radiation[6]. This type of modulation has been utilized in an assortment of utilizations viz. mode locking, pulse shaping, optical beam deflection, display, processing of information, and impression of information onto optical pulses *etc*.[7,8]. The acousto-optic modulators are based on the principle of laser-acoustic phonon (or low frequency electromagnetic wave) interaction in a nonlinear crystal. The modulation of electromagnetic waves by surface-acoustic waves is an ongoing field of research due to their potential applications in the field of communication devices[9,10].

The issue of engendering of laser beam in a nonlinear crystal in the presence of an acoustic strain field is additionally an ongoing field of research

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especially when modulation of light beam is created by sound waves. In addition, externally applied electric/magnetic fields add new dimensions to the modulation process[11]. The modulation of an electromagnetic wave can be made with respect to amplitude, frequency and phase. Out of these, amplitude modulation is one of the oldest form of modulations. In numerous complex modulation schemes, amplitude modulation is, in general, encountered as a preliminary step. In optical communication systems, a major issue is to build up an efficient technique for modulation as well as demodulation of electromagnetic waves. This often makes for maximum simplicity and cost efficient, particularly at low power outputs.

It is a well-known fact that, an unmodulated coherent electromagnetic wave while propagating through a nonlinear crystal with periodically varying parameters gets amplitude modulated. This periodic variation in propagation parameters may be induced by the propagation of an acoustic wave in plasmas[12]. The propagation of an acoustic wave in plasmas leads to the periodic variation in its electron density, which further leads to the modulation of coherent electromagnetic wave at the acoustic wave frequency.

The issues of amplitude modulation and demodulation of a coherent electromagnetic wave, due to their vast technological potential applications have been studied theoretically in a variety of nonlinear media[13,14]. Out of enormous number of nonlinear crystals, III-V semiconductor crystals provide a compact and more affordable medium to model modulation interactions. In addition, they offer extensive adaptability due to their prompt importance to issues of optical communication systems. This is due to:

(i) the enormous number of charge carriers (electrons/holes) in doped semiconductors show accessibility of free carrier states and the photo-generation of carriers,
(ii) the relaxation time of charge carriers can be very much constrained by materials design,
(iii) observation of large nonlinear optical susceptibilities in the vicinity of band-gap resonant transitions,
(iv) the nonlinear optical properties of doped semiconductors can be effectively adjusted by utilization of external electric/magnetic fields,
(v) semiconductor devices may operate either at normal incidence or in wave waveguides, and
(vi) semiconductor devices are integrable with other optoelectronic components.

Mathur and Sagoo[15] first predicted the modulation of microwave while propagating through a piezoelectric active semiconductor medium duly irradiated by an acoustic wave. Sen and Kaw[16] reported the modulation of a laser beam produced due to certain plasma effects in semiconductor crystals. Neogi[17] studied the issue of acousto-optic modulation of microwaves in piezoelectric semiconductors. Sharma and Ghosh[18] studied modulation interaction between co-propagating high frequency laser (pump) wave and acoustic wave and consequent amplification (steady-state and transient) of the modulated waves in a magnetized piezoelectric semiconductor medium. They performed the analysis under non-dispersive regime of the acoustic mode and found that the transient gain coefficient diminishes very rapidly if one chooses the pump pulse duration beyond the maximum gain point. Considering that the origin of modulation interaction lies in third-order optical susceptibility of the medium, Ghosh and Rishi[19] studied acousto-optic modulation in magnetized diffusive semiconductors and observed enhancement in gain coefficient of modulated beam in the presence of an external dc magnetic field in heavily doped regime. Using coupled mode theory, Nimje *et.al*.[20] studied hot carrier effects on diffusion induced modulation instability in magnetized semiconductors and observed that the heating effect reduces the required threshold amplitude of wave and enhances steady-state as well as transient gain of the generated acoustic mode. By considering the optical modulation as a four-wave interaction process, Malviya *et.al*.[21] studied acousto-optic modulation in ion implanted semiconductors having strain dependent dielectric constant and observed that the presence of colloidal grains plays an effective role in changing the threshold intensity and gain constant of modulated wave.

It shows up from above literature that an enormous number of endeavors have been made to examine the impacts of material parameters, externally applied fields and carrier heating induced by laser beam on amplitude modulation of a laser beam at acoustic wave frequency in semiconductor crystals. In these investigations, the dependence of threshold value of pump field for the onset of modulation amplification and gain coefficient of modulated wave on material parameters, externally applied magnetic field, *etc*.

have been reported in amplitude modulation of a laser beam by acoustic phonons in semiconductor crystals. Yet, the modulation index of optical amplitude modulated wave, being a significant parameter, has been not determined analytically. This demands more explanatory efforts are needed in the study of amplitude modulation of a coherent electromagnetic wave in magnetized doped III-V semiconductor crystals where periodic variation in propagation parameters have been induced by the propagation of an acoustic phonon mode.

Here, it should be worth pointing out that in most cases of investigation of various amplitude modulation mechanisms, the nonlocal effects, for example, diffusion of the excitation density responsible for the nonlinear refractive index change has been completely disregarded. The study of reflection and transmission of Gaussian beam incident upon an interface that separates a linear and nonlinear diffusive media has stimulated the idea to include diffusion in computation of nonlinear electromagnetic wave interaction in bulk and nonlinear-nonlinear interfaces[22,23]. It has been discovered that the increased diffusion makes light transmission more difficult and tends to wash out the local equilibrium of the equivalent potential representing unstable and stable transverse electric nonlinear surface waves[24]. The high mobility of excited charge carriers makes diffusion effects particularly relevant in semiconductor technology as they travel significant distances before recombination. Hence the diffusion of carrier is anyway expected to have a strong influence on the nonlinearity of the medium and hence the modulation phenomenon particularly in high mobility semiconductors, viz. III-V compound semiconductors, in which carriers can easily be moved. In this manner, the consideration of carrier diffusion in theoretical studies of modulation phenomenon is by all accounts significant from both the fundamental and application perspectives and thus attracted attention of researchers over the most recent few decades[25-27].

Inspired by above discussion, in the present paper, an analytical investigation of the amplitude modulation and demodulation of a coherent electromagnetic wave in III-V semiconductors has been made. The impact of carrier diffusion on this nonlinear interaction of the pump beam with semiconductor crystal adds a new dimension to the investigation. The intense pump beam generates an acoustic wave within the semiconductor crystal via electrostrictive mechanism and induces an interaction between the free charge carriers (through electron plasma wave) and the acoustic phonons (through material vibration). In semiconductor crystals, a part of mechanical energy of vibrations is in the form of electrical energy; the acoustic wave is then accompanied by an electromagnetic wave. Consequently one may anticipate strong interaction between acoustic wave and the pump wave in semiconductor crystals. This interaction leads to generation of a strong space charge field that modulates the pump wave. In this way, the applied pump wave and generated acoustic wave in an electrostrictive modulator can produce amplitude modulation and demodulation effect at acoustic wave frequency. From the available literature, it has been found that the application of magnetic field is favorable for the phenomenon under study[18-20]. Numerical estimations have been made with a set of data appropriate for a III-V semiconductor crystal (InSb) duly irradiated by a frequency doubled CO_2 laser to build up the legitimacy of the present work.

2 Theoretical Formulations

For theoretical formulation of modulation index of amplitude modulated pump beam in n-type doped semiconductor crystal, we have considered the hydrodynamic model of homogeneous semiconductor plasma of infinite extent at liquid nitrogen temperature (77K). At this specific temperature, the semiconductor medium empowers one to replace the streaming electrons with an electron fluid described by a few macroscopic parameters, for example, average velocity, average carrier density and so forth. This replacement simplifies the present analysis, without any loss of significant information. However, it restricts the analysis to be valid only in the limit (i.e. $k_a l \ll 1$, where k_a is the acoustic wave number and l is the carrier mean free path).

We consider the semiconductor crystal immersed in a static magnetic field $\vec{B}_0 = \hat{z}B_0$ (pointing along *z*-axis), which is illuminated by an intense pump wave $(\omega_0, \vec{k}_0)$. The pump wave produces density perturbations in the crystal. Here, the low frequency perturbations are assumed to be due to acoustic wave $(\omega_a, \vec{k}_a)$ generated through acoustic polarization in the crystal. In the present analysis, the dependence of field quantities assumed to vary as: $\exp[i(\omega_j t - k_j x)]$.

Due to electrostrictive potential fields accompanying the acoustic wave, the electron concentration oscillates at the acoustic wave frequency ω_a. The pump field then gives rise to a transverse current density at frequencies ω_0 and ($\omega_0 \pm p\omega_a$), where ω_0 is the pump wave frequency and p is an integer. The transverse current densities produced at frequency ω_0 is known as original (un-modulated) pump wave frequency, while those at ($\omega_0 \pm p\omega_a$) are known as side band current densities. These side band current densities produce side band electric field vectors and consequently the pump wave gets modulated. As we are interested only in first-order side bands ($\omega_0 \pm \omega_a$) with $p = 1$, the higher order side bands with $p \geq 2$, being off-resonant have been neglected. In the following analysis the side bands have been represented by the suffixes $\pm$, where + representing the mode propagating with frequency ($\omega_0 + \omega_a$) and − representing the mode propagating with frequency ($\omega_0 - \omega_a$).

The time varying pump electric field E_0 produce density perturbations and is thus capable of deriving acoustic waves in the semiconductor crystal. Let $u(x,t)$ be the deviation of a point x from its equilibrium position, so that the strain along the direction of pump wave is $\frac{\partial u}{\partial x}$. If E_1 is the derived space charge field then the net electrostrictive force in the positive *x*-direction acting on a unit volume can be expressed as[28]:

$$F = \frac{\gamma}{2}\frac{\partial}{\partial x}(E_0 E_1^*), \quad \dots (1)$$

where γ is a phenomenological constant describing the change in the optical dielectric constant and is generally known as electrostrictive coefficient.

Consequently, the equation of motion for $u(x,t)$ in the electrostrictive semiconductor crystal is given by:

$$\frac{\partial^2 u}{\partial t^2} - \frac{C}{\rho}\frac{\partial^2 u}{\partial x^2} + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{F}{\rho}$$

$$= \frac{\gamma}{2\rho}\frac{\partial}{\partial x}(E_0 E_1^*), \quad \dots (2)$$

where ρ is the mass density of crystal. C represents the elastic stiffness constant of the crystal such that the shear acoustic speed is given by $v_a = \sqrt{\frac{C}{\rho}}$. The term $2\Gamma_a \frac{\partial u}{\partial t}$ is introduced phenomenologically to include the acoustic damping, in which $\Gamma_a (\approx 10^{-11}\,\mathrm{SI\,units})$[29] is the phenomenological damping parameter of acoustic wave. The symbol "*" over a quantity represents its complex conjugate.

The acoustic wave generated due to electrostrictive property of the semiconductor crystal modulates the dielectric constant and gives rise to a nonlinear induced polarization given by:

$$P_{es} = -\gamma E_0 \frac{\partial u^*}{\partial x} \quad \dots (3)$$

In Eq. (3), the interaction of electrons among themselves and with nuclei of the atoms has been neglected by assuming the pump wave frequency very high as compared to the frequency of the motion of electrons in the medium. Thus, in the presence of electrostrictive polarization P_{es}, the electric displacement is simply given by[30]:

$$D = \varepsilon E_1 + P_{es}, \quad \dots (4)$$

where ε is the permittivity of the semiconductor medium and E_1 is the space charge field determined by the Poisson equation as:

$$\frac{\partial E_1}{\partial x} = -\frac{n_1 e}{\varepsilon} + \frac{\gamma}{\varepsilon}\frac{\partial^2 u^*}{\partial x^2} E_0 \quad \dots (5)$$

In order to compute the perturbed electron concentration n_1 in *n*-type semiconductor in the presence of electrostrictive coupling, we employ Eqs (2) and (5) and obtained the perturbed electron concentration as:

$$n_1 = -\frac{2\varepsilon u\rho}{e\gamma E_0}\left[\omega_a^2 + 2i\Gamma_a\omega_a - k_a^2 v_a^2\left(1 - \frac{\gamma|E_0|^2}{2\varepsilon C}\right)\right], \quad \dots (6)$$

where $\frac{\gamma|E_0|^2}{2\varepsilon C}$ is the dimensionless constant known as electrostrictive coupling coefficient due to the electrostrictive interaction in the medium under study.

The electron momentum transfer equation including diffusion effect is given as:

$$\frac{\partial \vec{v}_j}{\partial t}+(\vec{v}_0.\nabla)\vec{v}_j+\nu\vec{v}_j=-\frac{e}{m}[\vec{E}_j+(\vec{v}_j\times\vec{B}_0)]-\nu D\frac{\nabla(\vec{n}_j)}{n_j} \quad \dots (7)$$

The above equation describes the motion of electrons of effective mass m and charge e in the crystal under the influence of the electric fields associated with the pump and side band modes. Here, the subscript j stands for 0 (un-modulated), + (upper side band) and − (lower side band) modes. ν is phenomenological collision frequency of electrons. n_j represents the perturbed and unperturbed electron densities. D is the carrier diffusion coefficient which may be expressed (using Einstein's relation) as:

$$D=\frac{k_B T_e \mu_e}{e}, \quad \dots (8)$$

in which k_B, T_e and μ_e represent the Boltzmann constant, electron temperature, and electron mobility respectively.

In Eq. (7), the pump magnetic field has been neglected by assuming that the pump wave frequency is comparable to electron plasma frequency of the medium.

The momentum transfer equation [Eq. (7)] may be used to obtain the components of oscillatory electron fluid velocity in the presence of fields of the pump wave (E_0) and the side band modes ($E_\pm$). By linearizing Eq. (7) these velocity components may be obtained as:

$$v_{jx}=-\frac{e}{m}\frac{(\nu-i\omega_j)E_j}{[\omega_c^2+(\nu-i\omega_j)^2]}\left(1+\frac{\nu Dk^2}{\omega_p^2}\right) \quad \dots (9)$$

and

$$v_{jy}=-\frac{e}{m}\frac{\omega_c E_j}{[\omega_c^2+(\nu-i\omega_j)^2]}\left(1+\frac{\nu Dk^2}{\omega_p^2}\right), \quad \dots (10)$$

in which $\omega_c=\frac{e}{m}B_0$ is the electron-cyclotron frequency, and $\omega_p=\left(\frac{n_0e^2}{m\varepsilon}\right)^{1/2}$ is the electron plasma frequency.

The total transverse current density in n-type semiconductor medium is given by:

$$\vec{J}_{total}=\sum_j n_0 e\vec{v}_j+\sum_j ne\vec{v}_0\exp[i(\omega_j t-k_a x)], \quad \dots (11)$$

where the first term $\sum_j n_0 e\vec{v}_j$ represents the total current density generated due to pump wave and the second term $\sum_j ne\vec{v}_0\exp[i(\omega_j t-k_a x)]$ represents the total current density generated due to the interaction of the pump wave with acoustic wave.

The total electric field E_{total} associated with upper and lower side bands is given by:

$$E_{total}=E_\pm(-ikx)+E_\pm\exp[-i(k\pm k_a)x] \quad \dots (12)$$

In order to obtain the modulation indices of the modulated side band modes, we employ the general wave equation, which under the chosen configuration is given by:

$$\frac{\partial^2\vec{E}_{total}}{\partial x^2}-\mu\varepsilon\frac{\partial^2\vec{E}_{total}}{\partial t^2}-\mu\frac{\partial\vec{J}_{total}}{\partial t}=0, \quad \dots (13)$$

where μ is the permeability of the semiconductor medium.

Using Eqs (9), (10), (11) and (13) in the general wave equation [Eq. (13)], we obtain the expressions for modulation indices as:

$$\frac{E_\pm}{E_0}=\frac{-ie\mu u\omega_0}{\beta m}\frac{(\nu-i\omega_0)(\gamma k_a^2|E_0|^2+4i\varepsilon\rho\Gamma_a\omega_a)}{(\omega_c^2-\omega_0^2+\nu^2)(k_a^2\pm 2kk_a)}\left(1+\frac{\nu Dk^2}{\omega_p^2}\right), \quad \dots (14)$$

in which $\beta=\gamma E_0$.

In deriving above relation, we have assumed $\exp(-ikx)<<1$. This approximation is justifiable as $k\approx 10^7$ to 10^8 m^{-1} and for very high values of x, $\exp(-ikx)$ will remain negligible in comparison to unity.

Rationalizing Eq. (14), we obtain the real parts of modulation indices as:

$$\frac{E_\pm}{E_0}=\frac{-ie\mu u\omega_0}{\beta mk_a}\frac{[k_a^3\gamma|E_0|^2\,\omega_0(\omega_c^2-\omega_0^2-\nu^2)-4\varepsilon\rho v_a\nu\Gamma_a(\omega_c^2+\nu^2+\omega_0^2)]}{[(\omega_c^2-\omega_0^2+\nu^2)^2+4\nu^2\omega_0^2](k_a\pm 2k)}\left(1+\frac{\nu Dk^2}{\omega_p^2}\right) \quad \dots (15)$$

Equation (15) reveals that the magnitudes of the modulation indices in semiconductor plasma are influenced by acoustic wave number k_a, phenomenological damping parameter of acoustic wave Γ_a , carrier diffusion coefficient D, doping concentration n_0 (via electron-plasma frequency ω_p), and externally applied magnetic field B_0 (via electron-cyclotron frequency ω_c).

3 Results and Discussion

In this section the amplitude modulation and demodulation of a coherent electromagnetic wave due to electrostrictive interaction has been analyzed in a magnetized doped III-V semiconductor. Using Eq. (15), the modulation indices of plus (E_+ / E_0) and minus (E_- / E_0) mode has been plotted with respect to magnetic field B_0 (in terms of electron-cyclotron frequency ω_c) in the presence ($\Gamma_a \neq 0$) and absence ($\Gamma_a \neq 0$) of phenomenological damping parameter for a representative n-InSb semiconductor crystal duly irradiated by 10.6 µm pulsed CO_2 laser at 77 K. The physical parameters used are[31]:

$m = 0.0145 m_0$ (m_0 being the free mass of electron), $\varepsilon = 15.8$, $\gamma = 5\times10^{-10}$ s^{-1}, $\Gamma_a = 2\times10^{10}$ s^{-1}, $\rho = 5.8\times10^3$ kg m^{-3}, $\omega_a = 2\times10^{11}$ s^{-1}, $v_a = 4\times10^3$ m s^{-1}, $\nu = 4\times10^{11}$ s^{-1} and $\omega_0 = 1.78\times10^{14}$ s^{-1}.

We have analyzed Eq. (15) under two different regimes of wave number, i.e.

(i) $k_a > 2k$, and (ii) $k_a < 2k$.

Case (i): $k_a > 2k$

In this regime of wave number, the side band modes ($E_\pm$) are in phase with the pump wave in the plotted range of electron-cyclotron frequency. These in-phase side bands interact with the pump wave to generate the modulated acoustic wave. Hence, one always gets modulation in the wave regime $k_a > 2k$. Moreover, the modulation indices of both the side band modes are found to be a maximum at a particular value of magnetic field when electron-cyclotron frequency is in resonance with pump wave frequency ($\omega_c \approx \omega_0$).

The authors further divide the discussion into two parts:

(a) ***In the presence of phenomenological damping parameter*** **($\Gamma_a \neq 0$)**

The nature of dependence of modulation indices of plus (E_+ / E_0) and minus (E_- / E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) are depicted in Figs. 1 and 2, respectively. In both the figures, two curves are drawn; one in the presence of diffusion ($D \neq 0$) and the other in the absence of diffusion ($D = 0$) of charge carriers. It can be observed that the modulation indices of both the plus (E_+ / E_0) and minus (E_- / E_0) modes increase with electron-cyclotron frequency up to $\omega_c < \omega_0$. At a particular value of magnetic field ($B_0 = 14.2$ T), when electron-cyclotron frequency is in resonance with pump wave frequency ($\omega_c \approx \omega_0$), the modulation indices yield a maximum value. On further increasing the magnetic field ($\omega_c > \omega_0$), the modulation indices decrease abruptly. A comparison between the two curves reveals that the presence of carrier diffusion enhances the modulation indices of both the modes.

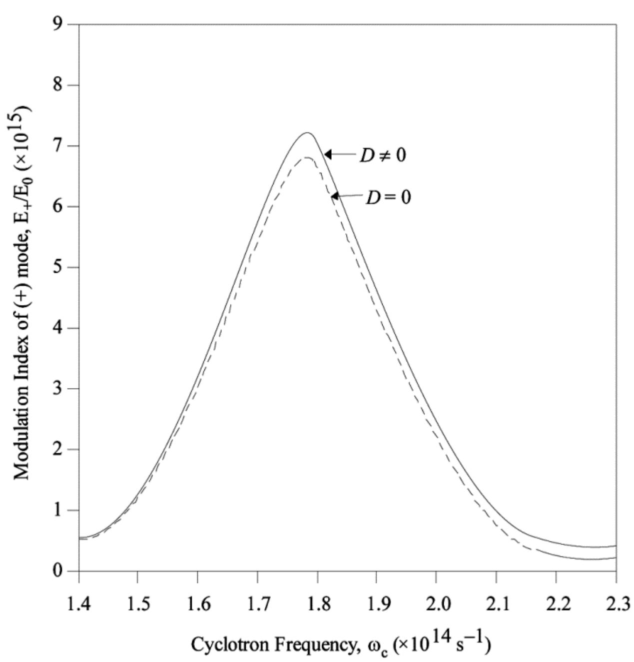

Fig. 1 — Nature of dependence of modulation index of plus mode E_+ / E_0 (for $k_a > 2k$ and $\Gamma_a \neq 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

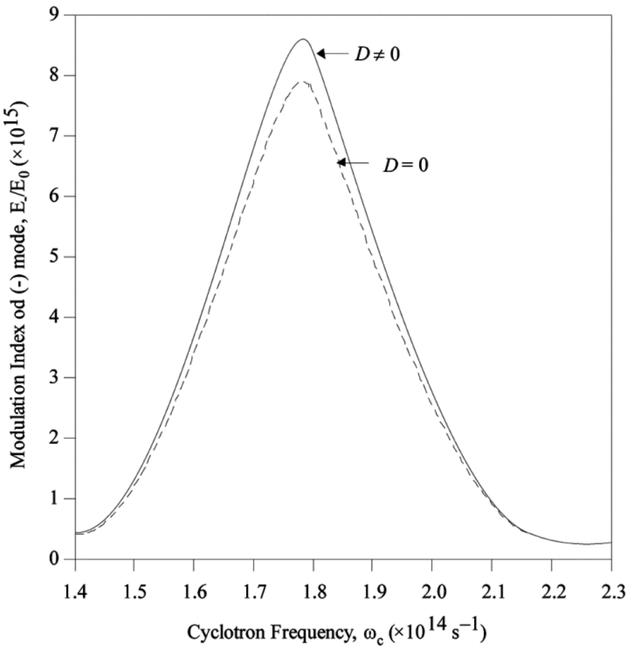

Fig. 2 — Nature of dependence of modulation index of minus mode E_-/E_0 (for $k_a > 2k$ and $\Gamma_a \neq 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

(b) ***In the absence of phenomenological damping parameter*** **($\Gamma_a = 0$)**

For the case when $\Gamma_a = 0$, Eq. (15) becomes

$$\frac{E_\pm}{E_0} = \frac{-ie\mu u\omega_0}{\beta m k_a}\frac{k_a^3\gamma|E_0|^2\omega_0(\omega_c^2-\nu^2-\omega_0^2)}{[(\nu^2+\omega_c^2-\omega_0^2)^2+4\nu^2\omega_0^2](k_a\pm 2k)}\left(1+\frac{\nu D k^2}{\omega_p^2}\right) \qquad \ldots (16)$$

For this case, the nature of dependence of modulation indices of plus (E_+/E_0) and minus (E_-/E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) are depicted in Figs. 3 and 4, respectively. In both the figures, again two curves are drawn; one in the presence of diffusion ($D \neq 0$) and the other in the absence of diffusion ($D = 0$) of charge carriers. It can be observed that for weak magnetic fields ($\omega_c < \omega_0$), both the side bands are in phase with pump wave. Hence both the upper and lower side bands get modulated in the absence of phenomenological parameter when electron-cyclotron frequency is smaller than pump wave frequency. At a particular value of magnetic field ($B_0 = 14.2$ T), when electron-cyclotron frequency is in resonance with

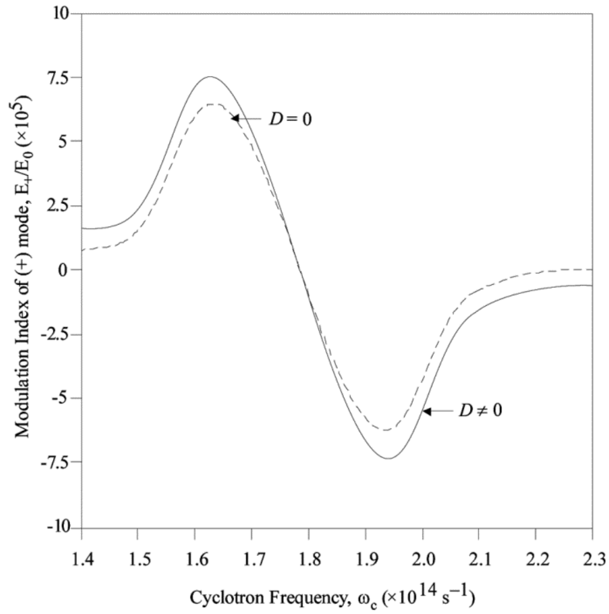

Fig. 3 — Nature of dependence of modulation index of plus mode E_+/E_0 (for $k_a > 2k$ and $\Gamma_a = 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

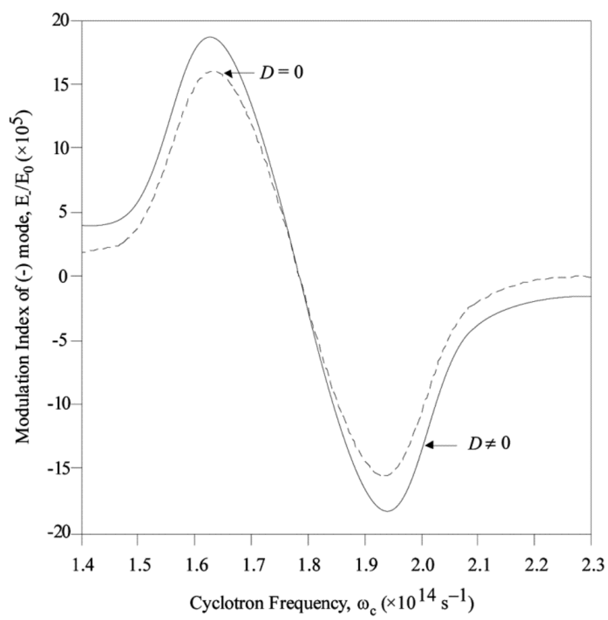

Fig. 4 — Nature of dependence of modulation index of minus mode E_-/E_0 (for $k_a > 2k$ and $\Gamma_a = 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

pump wave frequency ($\omega_c \approx \omega_0$), modulation indices of both the modes become zero (i.e. the pump electromagnetic wave gets un-modulated). For strong magnetic fields ($\omega_c > \omega_0$), both the modulation indices are negative representing that both the side bands go out of phase with pump wave and hence get demodulated. This nature may be explained if one may express the modulation indices in the form $(E_\pm / E_0) = -R$. One may introduce the negative sign in phase factor as: $(E_\pm / E_0) = -R\exp(i\pi)$. This indicates that a phase difference of π has been introduced between the modulated side band waves and the pump wave or the two waves are out of phase. Consequently, the modulated side band waves again interact with the pump wave thus producing the demodulated acoustic wave. Hence the phenomenon of demodulation takes place. Thus, the authors may conclude that in the regime of acoustic wave $k_a > 2k$, when $\omega_c > \omega_0$ and $\Gamma_a = 0$, the process of demodulation takes place. A comparison between the two curves reveals that the presence of carrier diffusion increases the modulation indices but decreases the demodulation indices of the side band modes. Moreover, a comparison between results of Figs. 3 and 4 reveals that the modulation index of minus mode is always greater than that of the plus mode.

To appreciate the role of phenomenological damping parameter (Γ_a) on modulation/demodulation process, the nature of dependence of modulation indices of plus (E_+ / E_0) and minus (E_- / E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) when $k_a < 2k$ are depicted in Figs. 5 and 6, respectively. In both the figures, two curves are drawn; one in the presence ($\Gamma_a \neq 0$) and the other in the absence ($\Gamma_a = 0$) of phenomenological damping parameter. It can be observed from these figures that when $\Gamma_a \neq 0$, the modulation indices of both the plus (E_+ / E_0) and minus (E_- / E_0) modes increase with electron-cyclotron frequency up to $\omega_c < \omega_0$, yield a maximum value when $\omega_c \approx \omega_0$ and decreases abruptly when $\omega_c > \omega_0$. Thus, in the presence of damping parameter, the modulation indices of both the side band modes are positive representing that the pump wave gets modulated as the side bands are always in phase with pump wave. But when $\Gamma_a = 0$, the modulation indices of both the plus (E_+ / E_0) and

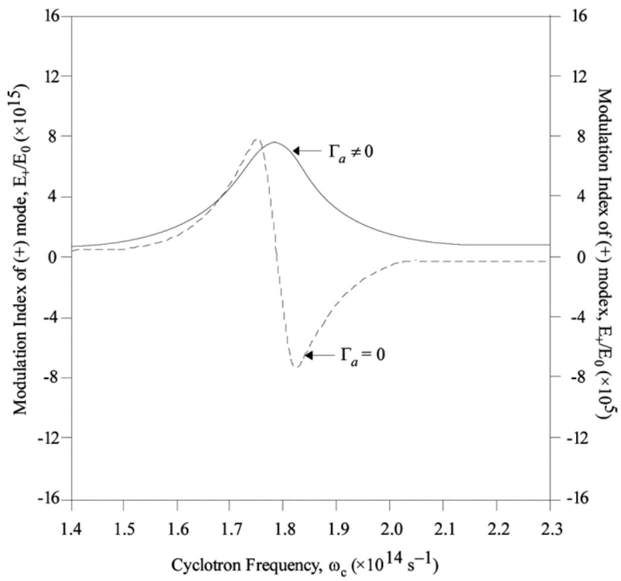

Fig. 5 — Nature of dependence of modulation index of plus mode E_+ / E_0 (for $k_a > 2k$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $\Gamma_a = 0$ and $\Gamma_a \neq 0$.

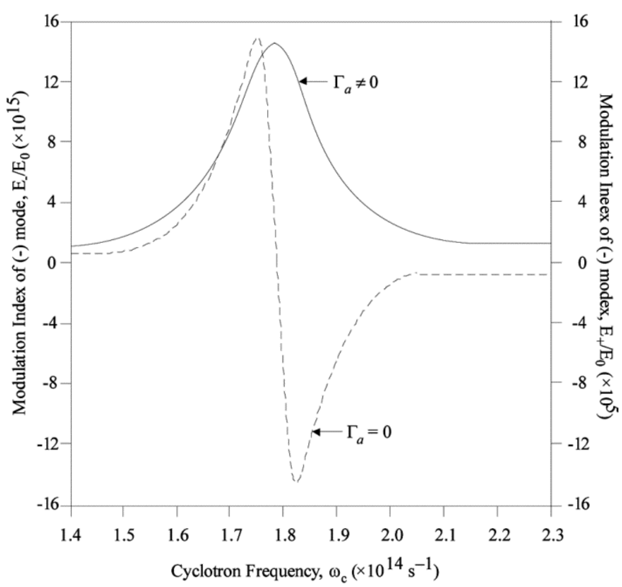

Fig. 6 — Nature of dependence of modulation index of minus mode E_- / E_0 (for $k_a > 2k$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $\Gamma_a = 0$ and $\Gamma_a \neq 0$.

minus (E_- / E_0) modes are positive (representing modulated state) when $\omega_c < \omega_0$, becomes zero (representing un-modulated state) when $\omega_c \approx \omega_0$ and becomes negative (representing demodulated state) when $\omega_c > \omega_0$. A comparison between results of Figs. 5 and 6 again reveals that the modulation index of minus mode is always greater than that of the plus mode. Hence phenomenological damping parameter has a positive influence over this phenomenon when $k_a > 2k$.

Case (ii): $k_a < 2k$

The authors further divide the discussion into two parts:

(a) ***In the presence of phenomenological damping parameter*** **($\Gamma_a \neq 0$)**

The nature of dependence of modulation indices of plus (E_+/E_0) and minus (E_-/E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) are depicted in Figs. 7 and 8, respectively. In both the figures, two curves are drawn; one in the presence of diffusion ($D \neq 0$) and the other in the absence of diffusion ($D = 0$) of charge carriers. On one hand, it can be observed (Fig. 7) that the modulation index of plus mode increases with electron-cyclotron frequency up to $\omega_c < \omega_0$, yield a maximum value when $\omega_c \approx \omega_0 = 1.78 \times 10^4$ s^{-1} and decreases abruptly when $\omega_c > \omega_0$. The positive values of modulation index of plus mode represent the fact that the pump wave gets modulated as the upper side band is in phase with pump wave. On the other hand, it can be observed (Fig. 8) that the behavior of modulation index of minus mode is exactly opposite to that of plus mode. Hence the lower side band mode gets demodulated in this regime. A comparison between the two curves reveals that the carrier diffusion increases both modulation and demodulation indices.

(b) ***In the absence of phenomenological damping parameter*** ($\Gamma_a = 0$)

For this case, the nature of dependence of modulation indices of plus (E_+/E_0) and minus (E_-/E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) are depicted in Figs. 9 and 10, respectively. In both the figures, again two curves are drawn; one in the presence of diffusion ($D \neq 0$) and the other in the absence of diffusion ($D = 0$) of charge carriers. It can be observed that the amplitudes of the plus and minus side band modes are in phase with the pump wave under the conditions $\omega_c < (\omega_0^2 + \nu^2)^{1/2}$ and $\omega_c > (\omega_0^2 + \nu^2)^{1/2}$, respectively.

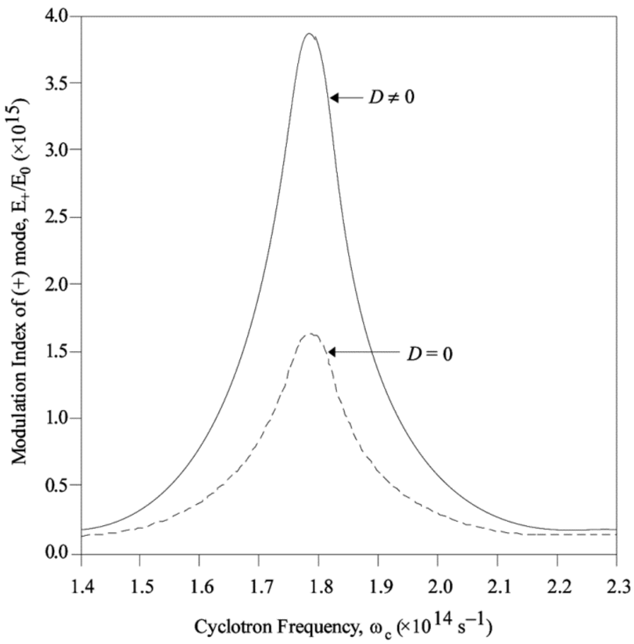

Fig. 7 — Nature of dependence of modulation index of plus mode E_+/E_0 (for $k_a < 2k$ and $\Gamma_a \neq 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

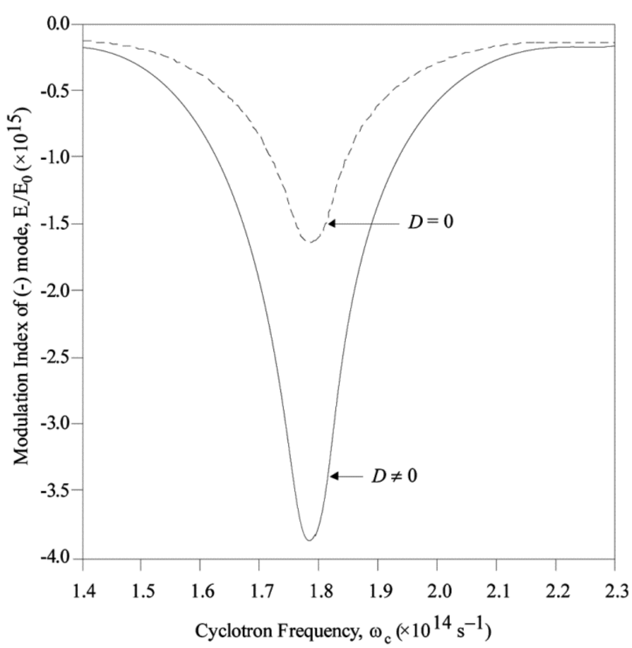

Fig. 8 — Nature of dependence of modulation index of minus mode E_-/E_0 (for $k_a < 2k$ and $\Gamma_a \neq 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

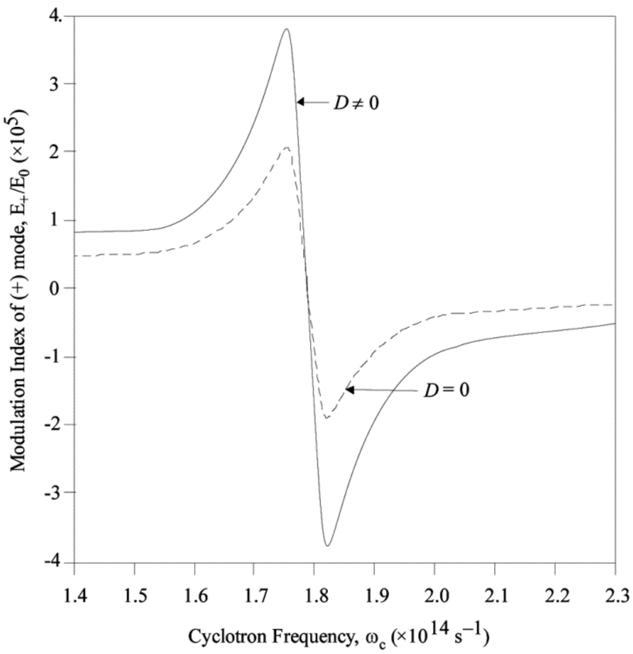

Fig. 9 — Nature of dependence of modulation index of plus mode E_+/E_0 (for $k_a < 2k$ and $\Gamma_a = 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

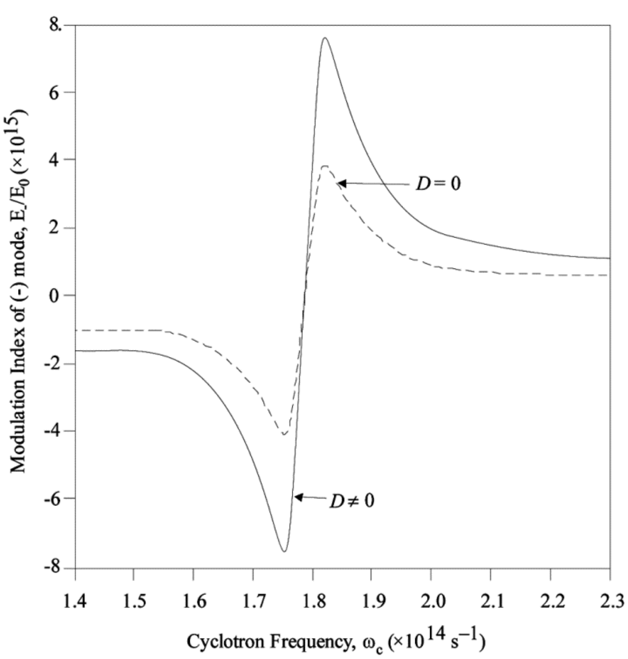

Fig. 10 — Nature of dependence of modulation index of minus mode E_-/E_0 (for $k_a < 2k$ and $\Gamma_a = 0$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $D = 0$ and $D \neq 0$.

At a particular value of electron-cyclotron frequency if plus mode gets modulated, the minus mode gets demodulated and vice versa. Hence both the side band modes have opposite nature in the wave number regime $k_a < 2k$. This is so because upper side band (+) mode gets modulated at low magnetic field when $\omega_c < (\omega_0^2 + \nu^2)^{1/2}$ whereas lower side band (-) mode gets modulated in the opposite condition. A comparison between the two curves reveals that the carrier diffusion always increases the modulation indices of both the plus and minus side band modes. At very high magnetic fields ($\omega_c >> \omega_0$), the amplitude modulation starts decreasing whereas demodulation starts increasing. This can directly be inferred from Eq. (16) where the magnitude of amplitude-ratio becomes inversely proportional to the square of the electron-cyclotron frequency when ω_c becomes quite larger than pump and electron-collision frequencies.

To appreciate the role of phenomenological damping parameter (Γ_a) on modulation/demodulation process, the nature of dependence of modulation indices of plus (E_+/E_0) and minus (E_-/E_0) modes on magnetic field B_0 (via electron-cyclotron frequency ω_c) when $k_a < 2k$ are depicted in Figs. 11 and 12, respectively. In both the figures, two curves are drawn; one in the presence ($\Gamma_a \neq 0$) and the other in the absence ($\Gamma_a = 0$) of phenomenological damping parameter. It can be observed that in the presence of damping, the plus mode is in phase (modulated state) whereas the minus mode is always out of phase (demodulated state) with pump wave in the regime of magnetic field considered. In the absence of damping, one observes both types of characteristics, i.e. modulation and demodulation for the plus and minus side band modes depending upon the relative magnitudes of electron-cyclotron frequency ω_c and pump frequency ω_0. For smaller values of magnetic field ($\omega_c < \omega_0$), the plus mode gets modulated whereas the minus mode gets demodulated. Whereas for higher values of magnetic field ($\omega_c > \omega_0$), the plus mode gets demodulated whereas the minus mode gets modulated. Particularly at $\omega_c = \omega_0$, both the plus and minus side band modes remains un-modulated.

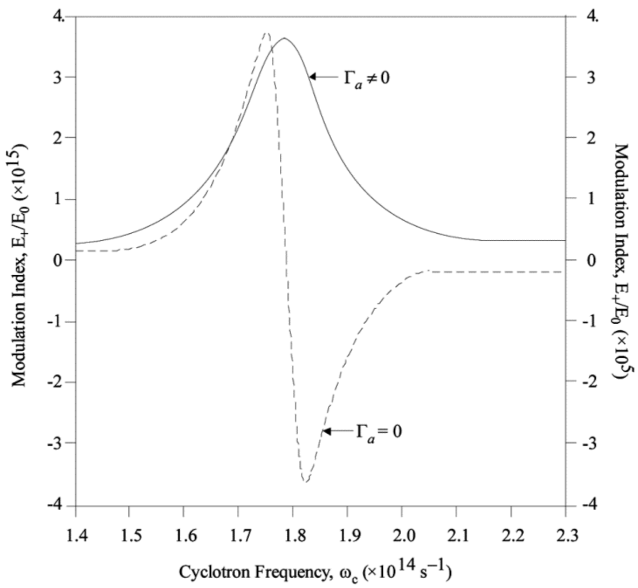

Fig. 11 — Nature of dependence of modulation index of plus mode E_+/E_0 (for $k_a < 2k$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $\Gamma_a = 0$ and $\Gamma_a \neq 0$.

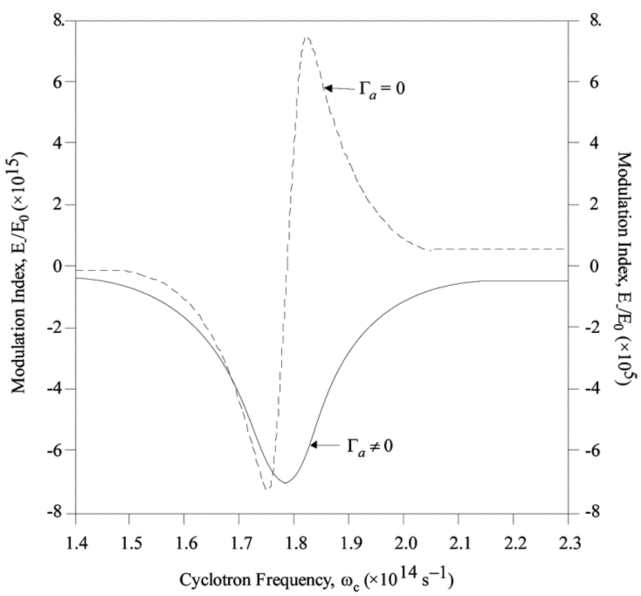

Fig. 12 — Nature of dependence of modulation index of minus mode E_-/E_0 (for $k_a < 2k$) on magnetic field B_0 (via electron-cyclotron frequency ω_c) for two different cases when $\Gamma_a = 0$ and $\Gamma_a \neq 0$.

From Eq. (16), a direct dependence of the ratio of pump frequency ω_0 and the acoustic intensity u is observable. However, in the vicinity of the wave number regime $k_a = 2k$, one may observe a singularity in the field ratio. Here, it should be worth mentioning that Lashmor-Davies[32] has explained the behaviour in this wave regime and accordingly, the decay instability takes place showing the absence of modulation. Using Eq. (16), the authors have compared the amplitude ratio for magnetized and un-magnetized semiconductor plasma as

$$\frac{(E_\pm/E_0)_{B_0\neq 0}}{(E_\pm/E_0)_{B_0=0}} = \frac{1-\dfrac{\omega_c^2}{(\omega_0^2+\nu^2)}}{1+\dfrac{\omega_c^2[\omega_c^2+2(\nu^2-\omega_0^2)]}{[(\nu^2-\omega_0^2)^2+4\omega_0^2\nu^2]}} \quad \ldots (17)$$

A close look at Eq. (17) reveals that the numerator is always less than one but denominator can become either greater than one (when $\omega_c^2 > 2(\omega_0^2+\nu^2)$) or less than one (when $\omega_c^2 < 2(\omega_0^2+\nu^2)$). Consequently, two cases may arise: (i) when $\omega_c^2 > 2\omega_0^2$, and (ii) $\omega_c^2 < 2\omega_0^2$.

Since $\nu << \omega_0$, the amplitude ratio at $\omega_c^2 > 2\omega_0^2$ after neglecting the electron-collision frequency becomes

$$\frac{(E_\pm/E_0)_{B_0\neq 0}}{(E_\pm/E_0)_{B_0=0}} = \frac{1-(\omega_c/\omega_0)^2}{1+(\omega_c/\omega_0)^4} \quad \ldots (18)$$

From Eq. (18), one may obtain the decrease in amplitude ratio due to higher magnetic fields (at $\omega_c^2 > 2\omega_0^2$). As $(\omega_c^2/\omega_0^2) > 2$, the numerator becomes negative. Hence, in this range of magnetic field (ω_c), the demodulation takes place.

In the range of magnetic field when $\omega_c^2 < 2\omega_0^2$, Eq. (17), reduces to

$$\frac{(E_\pm/E_0)_{B_0\neq 0}}{(E_\pm/E_0)_{B_0=0}} = \frac{1-(\omega_c/\omega_0)^2}{1-(2\omega_c^2/\omega_0^2)} \quad \ldots (19)$$

This ratio is always greater than one and it can be inferred that the modulation of the pump wave can be enhanced by increasing the magnetic field.

4 Conclusions

In the present study, using hydrodynamic model of semiconductor plasma the amplitude modulation and demodulation of a coherent electromagnetic pump wave in magnetized doped III-V semiconductors have been undertaken. Numerical estimations made for n-InSb-CO_2 laser system enables one to draw following conclusions:

(i) The hydrodynamic model of semiconductor-plasma has been successfully applied to study the effects of diffusion coefficient, damping parameter and external magnetostatic field (via electron-cyclotron frequency) on modulation indices of plus and minus side band modes in III-V semiconductor crystals.

(ii) The amplitude modulation/demodulation of a coherent electromagnetic wave by an acoustic wave can be easily achieved in electrostrictive III-V semiconductor plasma medium.

(iii) The damping plays a very important role in deciding the parameter range and selecting the side band mode, which will be modulated by the above-mentioned interaction.

(iv) The presence of carrier diffusion alters the result favorably. It always increases the value of modulation/demodulation indices for both the modes around resonance (when electron-cyclotron frequency ~ pump wave frequency).

(v) The consideration of a diffusive crystal with electrostrictive polarization possibly offers an interesting area for the purpose of investigations of different modulation interactions and one hopes to open a potential experimental tool for energy transmission and solid state diagnostics in electrostrictive diffusive crystals.

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