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Impact of hematocriton the flow of casson fluid in contact with jeffery fluid over a narrow pipe

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The flow of Casson fluid in contact with Jeffery fluid in a tube with a small diameter is concentrated in this paper. Casson fluid is considered in the central region and Jeffery fluid in the outer region. The governing equations are solved analytically and obtained expressions for velocity, flux, effective viscosity, central hematocrit, and mean hematocrit in dimensional form. The effect of many physical parameters on velocity, flux, effective viscosity, central hematocrit, and mean hematocrit is observed through graphs. Central hematocrit increases with an increase in yield stress and hematocrit and means hematocrit decreases with an increase in yield stress and hematocrit. The paper finds its applications in the flow of blood in narrow arteries, vines, and capillaries. The problem warrants further study on the flow of non-Newtonian two-fluid flows in a narrow tube.

Keywords: Effective viscosity, Casson fluid, Jeffery fluid, central hematocrit, mean hematocrit

1 Introduction

Now a day's diseases can be diagnosed and treatment can be monitored only through blood analysis. A specific blood test is required for detecting many diseases, which include biochemical or metabolic abnormalities. The volume of red blood cells that are present in the blood is called hematocrit and is measured in terms of percentages. Oxygen is transferred from lungs to tissues and carbon-di-oxide is transferred from tissues to lungs through red blood cells for expiration Mairbaurl[1]. The blood can be broken down into plasma volume and red cell volume. Therefore, blood has to be considered as two-fluid models. So two-fluid flows in tubes are very important for a better understanding of the behavior of blood in arteries and capillaries. Strait et al.[2], the effect of immiscible fluids in a capillary tube. A two-phase model in a narrow tube was analyzed by Sharan and Popel[3]. Recent research says that the researchers are giving importance to flow in arteries by[4-7].

In human lifeblood plays a vital role. Approximately 4-5 liters of blood is circulated by the heart to different parts of the body via different webs of vessels. The blood is pumped by the heart to diverse parts of the body through arteries and meandering into very narrower and denser networks of vessels. Santhosh and Radhakrishnamacharya[8] studied the Jeffrey fluid flow in Narrow tubes including magnetic effects. Blood past narrow vessels was investigated by Srivastava[9]. Siddiqui *et al.*[10] observed the movement of Casson liquid through a tube and found the effect of the Metachronal beating of cilia over it. Maqbool *et al.*[11] gave a closed-form solution by studying a simple non-Newtonian fluid past an inclined tube including the effect of cilia. Many authors are concentrating on the flows of Casson fluids.[12-16]

Misra and Ghosh[17] Studied on the flow of Casson fluid in a Narrow tube and also the impact of the side branch. The slip effect on the flow of Hershel-Bulkley fluid in a Narrow tube was investigated by Santhosh *et al.*[18]. A two-fluid model for Hershel- Bulkley fluid in a narrow tube was observed by Santhosh and Radhakrishnamacharya[19]. Siegfried[20] found the impact of effective viscosity on the flow of fluid in a circular cylinder. Effective viscosity of the spreading epithelia was analyzed by Blanch-Mercader *et al.*[21] Recently Naumov *et al.*[22] studied capillary hysteresis in two-fluid flows.

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Very few researchers concentrated on the stream of two-fluid models in a narrow pipe. The current paper is a concentration on the movement of Casson liquid in contact with Jeffery liquid through a narrow pipe. The equations governing the motion of the fluid were solved analytically and expressions for velocity flux, effective viscosity, hematocrit and mean hematocrit was obtained. The influence of many physical parameters was observed through graphs.

2 Mathematical Formulation of the Problem

The two-fluid, steady, laminar, fully developed and axisymmetric flow in a narrow tube with a uniform cross-section of the radius a is considered in this paper. The central region of the tube is occupied by Casson fluid whereas the outer region is occupied by Jeffery fluid. The central region is of radius b which includes the plug region of radius r_0 and outer region is of radius $a-b$ which is shown in Fig (1). To represent the two-dimensional fluid flow cylindrical coordinate system (r,z) is chosen, where r and z represent the radius and axis of the tube respectively.

The flow phenomenon is represented by the following governing equations [23 & 24].

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau)=P \quad \ldots (1)$$

Where $$(\tau)^{1/2}=\begin{cases}\mu(-\partial u/\partial r)^{1/2}+\tau_0^{1/2}, & \tau\ge\tau_0\\ 0, & \tau<\tau_0\end{cases} \quad \ldots (2)$$

and $P=-\dfrac{\partial p}{\partial z}$

Here u is the velocity, τ_0 represents the yield stress of the tube and μ is the consistency factor.

Let $\tau=\tau_1$ and $u=w_1$ in the outer region and $\tau=\tau_2$ $u=w_2$ in the central region. Then the flow of the fluid in the outer and central region is governed by the following equations:

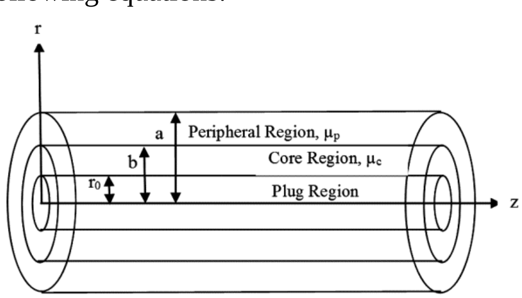

Fig. 1 — Physical Model.

Outer region

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_1)=P$$

$$\tau_1=-\frac{\mu_p}{1+\lambda_1}\left(\frac{\partial w_1}{\partial r}\right),\quad b\le r\le a \quad \ldots (3)$$

Central region

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_2)=P\ ,\quad r_0\le r\le b$$

$$(\tau_2)^{1/2}=\begin{cases}\mu(-\partial w_2/\partial r)^{1/2}+\tau_0^{1/2}, & \tau\ge\tau_0\\ 0, & \tau<\tau_0\end{cases} \quad \ldots (4)$$

The boundary conditions are given by

$$\tau_1=\tau_2 \quad at \quad r=b \quad \ldots (5)$$

$$\tau \;\; is\;\; finite\;\; at \quad r=0 \quad \ldots (6)$$

$$w_1=0 \quad at \quad r=a \quad \ldots (7)$$

$$w_1=w_2 \quad at \quad r=b \quad \ldots (8)$$

Using boundary conditions (5) and (6) equations (3) & (4) reduces to:

Outer region

$$\frac{\partial w_1}{\partial r}=-\frac{P\,\mathrm{r}(1+\lambda_1)}{2\mu_p}\ ,\quad b\le r\le a \quad (9)$$

Central region

$$\frac{\partial w_2}{\partial r}=-\frac{P}{2\mu_c}\left(r^{1/2}-r_0^{1/2}\right)^2\ ,\quad r_0\le r\le b \quad \ldots (10)$$

Solving equations (9) & (10) using the boundary conditions (7) & (8) we have,

$$w_1=\frac{P(1+\lambda_1)}{4\mu_p}\left(a^2-r^2\right)\;\; for\;\; b\le r\le a \quad \ldots (11)$$

$$w_2=\frac{P}{4\mu_c}\left(\frac{b^2-r^2}{2}+r_0(b-r)-\frac{4}{3}r_0^{1/2}(b^{3/2}-r^{3/2})\right)+\frac{P(1+\lambda_1)}{4\mu_p}(a^2-b^2)\;\; for\;\; r_0\le r\le b \quad \ldots (12)$$

As the fluid in the central region is Casson the velocity for plug region is obtained by taking $r=r_0$ in equation (12) and is given by

$$w_p=\frac{P}{2\mu_c}\left(\frac{b^2}{2}+r_0b-\frac{4}{3}r_0^{1/2}b^{3/2}-\frac{1}{6}r_0^{\,2}\right)+\frac{P(1+\lambda_1)}{2\mu_p}(a^2-b^2)\;\; for\;\; 0\le r\le r_0 \quad \ldots (13)$$

The flow flux in the outer and central region are respectively given by

$$F_p = 2\pi \int_b^a w_1 r \ dr \ , \quad b \le r \le a \qquad \dots (14)$$

$$F_c = 2\pi \left(\int_0^{r_0} w_p r \ dr + \int_{r_0}^{b} w_2 r \ dr \right), \ 0 \le r \le b \qquad \dots (15)$$

Using equations (11), (12) and (13) in equations (14) and (15) we have,

$$F_p = \frac{P\pi a^4 (1+\lambda_1)}{8\mu_p} \left(1-q^2\right)^2, \ b \le r \le a \qquad \dots (16)$$

and

$$F_c = \frac{P\pi a^4}{8\mu_p} \begin{bmatrix} 4\mu' q^4 \left(\frac{1}{4} + \frac{\tau_d}{3} - \frac{4}{7}\tau_d^{1/2} - \frac{11}{42}\tau_d^{\ 4} \right) \\ +2(1+\lambda_1)\left(1-q^2\right)q^2 \end{bmatrix} \qquad \dots (17)$$

$$, \ 0 \le r \le b$$

Where $\mu' = \frac{\mu_p}{\mu_c}, \ q = \frac{b}{a}, \ \tau_d = \frac{r_0}{b}$ … (18)

Where $q = 1 - \frac{\varepsilon}{a}$ is the non-dimensional central radius. Here $\varepsilon = 3.12\mu$ for 40% hematocrit, $\varepsilon = 3.60\mu$ for 30% hematocrit, and $\varepsilon = 4.67\mu$ for 20% hematocrit. [25 & 26].

Thus (16) and (17) the flux of the fluid flow in the tube is given by

$$F = F_p + F_c$$

$$= \frac{P\pi a^4}{8\mu_p} \begin{bmatrix} (1+\lambda_1)(1-q^4) + \\ 4\mu' q^4 \left(\frac{1}{4} + \frac{\tau_d}{3} - \frac{4}{7}\tau_d^{1/2} - \frac{11}{42}\tau_d^{\ 4} \right) \end{bmatrix} \qquad \dots (19)$$

Effective viscosity can be calculated by comparing (19) with Poiseuille's flow flux, which represented as below:

$$\mu_{eff} = \frac{\mu_p}{\gamma}$$

$$\gamma = 4\mu' q^4 \left(\frac{1}{4} + \frac{\tau_d}{3} - \frac{4}{7}\tau_d^{1/2} - \frac{11}{42}\tau_d^{\ 4} \right) + (1+\lambda_1)(1-q^4) \qquad \dots (20)$$

3 Performance of Mean Hematocrit

Hematocrit is defined as the percentage of RBC available in the human body. For an adult, it is estimated to be between 40-45 percent. The blood that flows in a small diameter tube contains a hematocrit of 45%. The total flow rate is 16 cc/sec, among which 12 cc/sec flows in the central region and 4 cc/sec flows in the outer region. 10 $mm^3$is the volume of blood cells accumulated in the central region and 6 mm^3 is the volume of blood cells present in the cell-free outer region.

The volume of the RBC that comes into the tube and goes out of the tube H is related to central hematocrit H_c [27]

$$HF = H_c F_c \qquad \dots (21)$$

Substituting F and Fc in equation (21) we have

$$H_{Nc} = \frac{H_c}{H} = 1 + \frac{(1+\lambda_1)(1-2q^2+q^4)}{H^*}$$

$$H^* = 4\mu' q^4 \left(\frac{1}{4} + \frac{\tau_d}{3} - \frac{4}{7}\tau_d^{1/2} - \frac{11}{42}\tau_d^{\ 4} \right) + 2(1+\lambda_1)\left(1-q^2\right)q^2 \qquad \dots (22)$$

where H_{Nc} is the normalized central hematocrit.

The mean hematocrit within the tube H_m is related to the central hematocrit H_c by

$$H_m \pi a^2 = H_c \pi b^2 \qquad \dots (23)$$

On simplification we get

$$H_{Nm} = \frac{H_m}{H} = \frac{H_c}{H} q^2 \qquad \dots (24)$$

where H_{Nm} is the normalized mean hematocrit.

Substituting equation (22) in equation (24) we have,

$$H_{Nm} = q^2 \left(1 + \frac{(1+\lambda_1)(1-2q^2+q^4)}{\chi} \right)$$

$$\chi = 4\mu' q^4 \left(\frac{1}{4} + \frac{\tau_d}{3} - \frac{4}{7}\tau_d^{1/2} - \frac{11}{42}\tau_d^{\ 4} \right) + 2(1+\lambda_1)\left(1-q^2\right)q^2 \qquad \dots (25)$$

4 Results and Discussion

To detect the abnormality in the body blood test is more essential as blood is the key factor to be infected first by any bacteria. In blood, the volume of red blood cells called hematocrit. Tracking the hematocrit is very important for a healthy human body, but it depends upon the nature of blood. So the current paper deals with the flow of Casson liquid in contact with Jeffrey liquid a model for blood in a narrow pipe. In this paper effective viscosity, hematocrit and mean hematocrit is obtained analytically. Effect of yield stress, Jeffery parameter, the viscosity of the fluid in central region and viscosity of the fluid in the outer region on effective viscosity, hematocrit and mean hematocrit were discussed through graphs which are shown in figures (2) - (15).

The change in effective viscosity for various physical parameters is depicted from the figures (2) - (6). From the figures (2) & (3), we observe that effective viscosity increases with an increase in yield stress and hematocrit. If the viscosity of the fluid in central region changes by fixing the viscosity of the fluid in the outer region, then how the effective viscosity is effected is noticed in figure (4). It is observed that the effective viscosity is more if the viscosity of the central fluid is more than the viscosity of the outer fluid. Also for fixed viscosity of the central fluid and for different values for viscosities of outer fluid the effective viscosity nature is studied through figure (5). Effective viscosity increases with an increase in viscosity of the outer fluid. From figure (6), it is noticed that the increment in the Jeffery

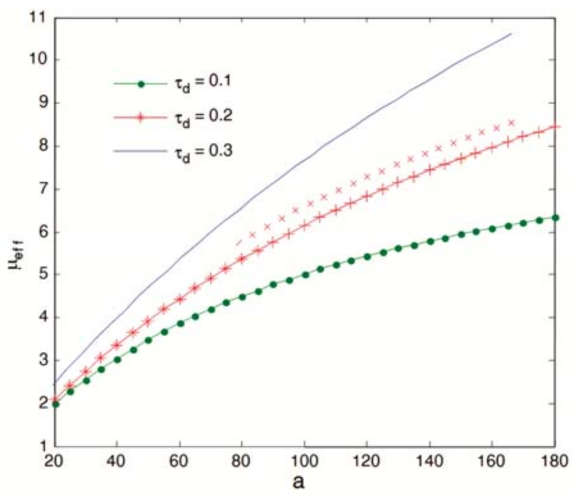

Fig. 2 — Effect of yield stress on μ_{eff} for fixed values of $H = 40\%,\ \mu_p = 1.2, \mu_c = 4.0, \lambda_1 = 0.1$

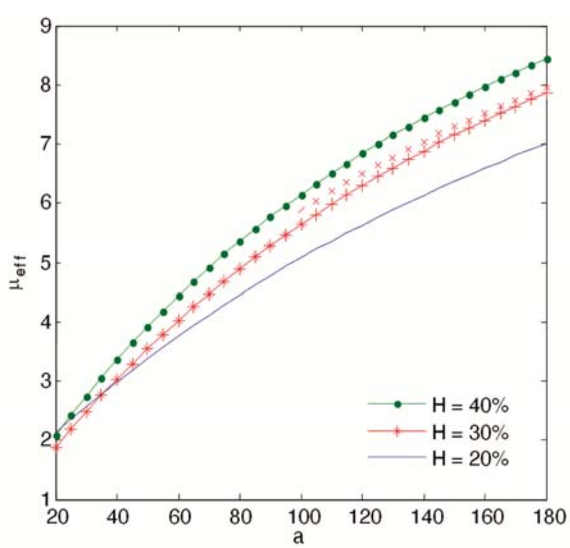

Fig. 3 — Effect of hematocrit on μ_{eff} for fixed values of $\tau_d = 0.2, \mu_p = 1.2cp, \mu_c = 4.0cp, \lambda_1 = 0.1$

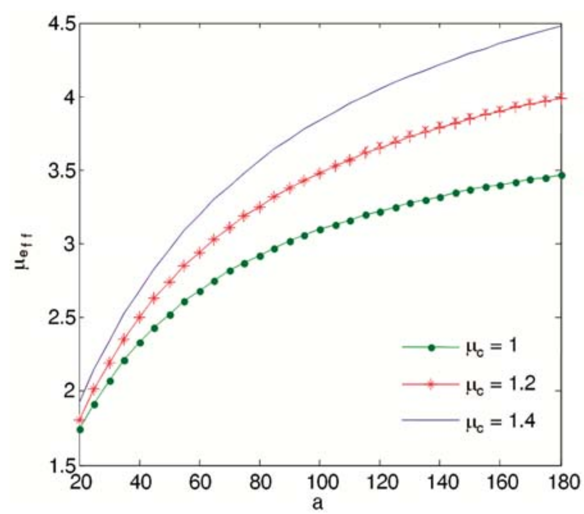

Fig. 4 — Effect of central viscosity on μ_{eff} for fixed values of $H = 40\%, \tau_d = 0.2, \mu_p = 1.2cp, \lambda_1 = 0.1$

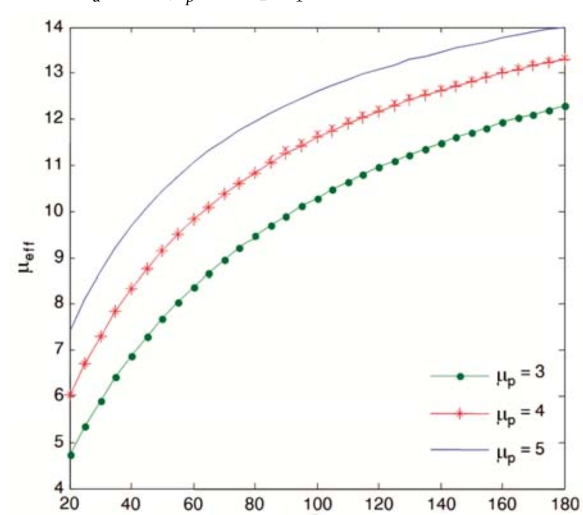

Fig. 5 — Effect of the viscosity of the Newtonian fluid on μ_{eff} for fixed values of $H = 40\%, \tau_d = 0.2, \mu_c = 4.0cp, \lambda_1 = 0.1$

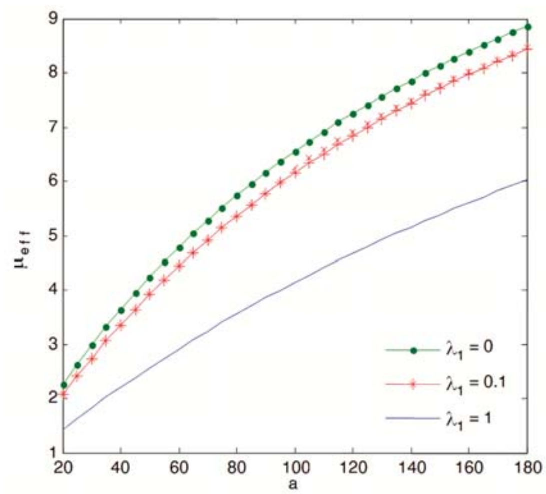

Fig. 6 — Effect of Jeffery parameter on μ_{eff} for fixed values of $H = 40\%, \tau_d = 0.2, \mu_c = 4.0cp, \mu_p = 1.2cp$

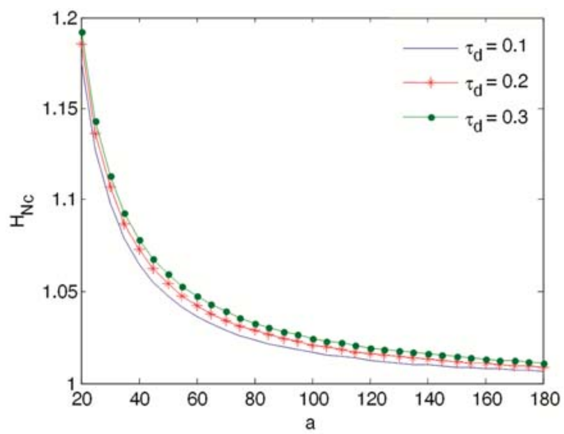

Fig. 7 — Effect of yield stress on central hematocrit for fixed values of $H = 40\%,\ \mu_p = 1.2cp, \mu_c = 4.0cp, \lambda_1 = 0.1$

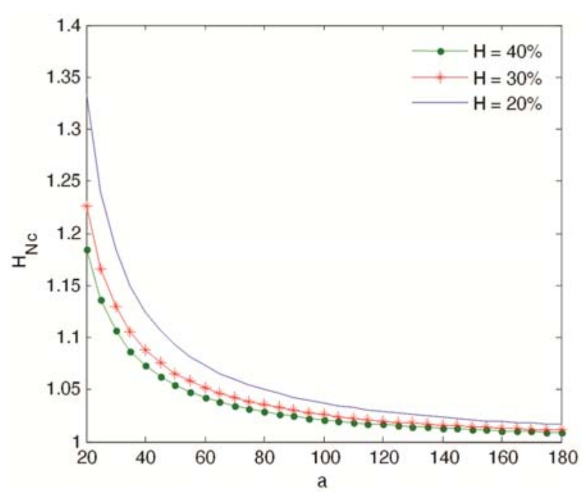

Fig. 8 — Effect of hematocrit on central hematocrit for fixed values of $\tau_d = 0.2, \mu_p = 1.2cp, \mu_c = 4.0cp, \lambda_1 = 0.1$

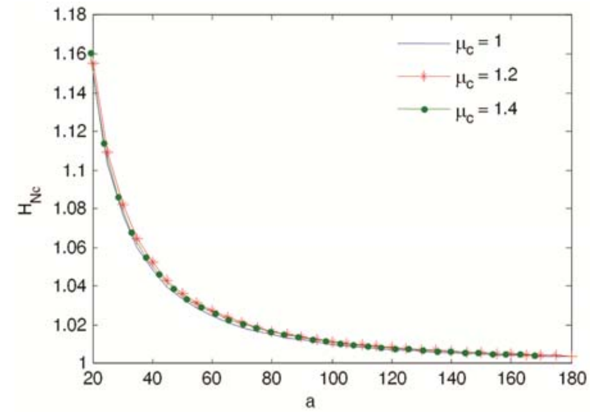

Fig. 9 — Effect of Casson fluid viscosity on central hematocrit for fixed values of $H = 40\%, \tau_d = 0.2, \mu_p = 1.2cp, \lambda_1 = 0.1$

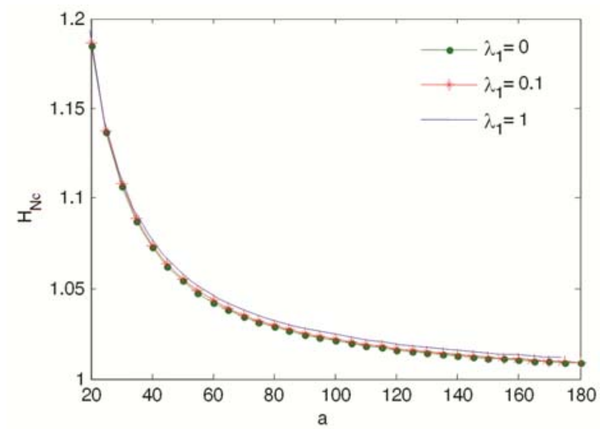

Fig. 10 — Effect of Jeffery parameter on central hematocrit for fixed values of $H = 40\%, \tau_d = 0.2, \mu_c = 4.0cp, \mu_p = 1.2cp$

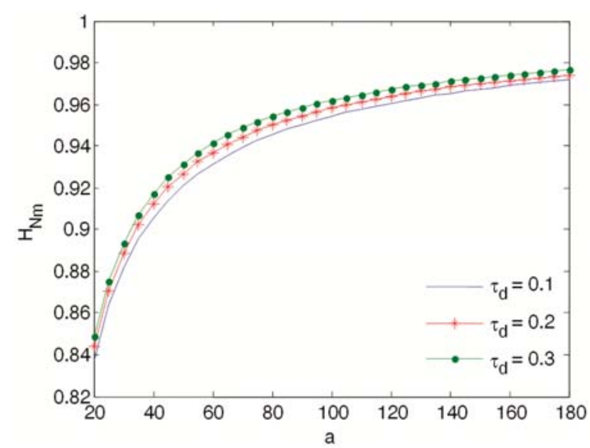

Fig. 11 — Effect of yield stress on mean hematocrit for fixed values $H = 40\%, \mu_p = 1.2cp, \mu_c = 4.0cp, \lambda_1 = 0.1$

parameter, increases the effective viscosity. From there it is found that Effective viscosity increases with the increase in either of the viscosities and Jeffery parameter.

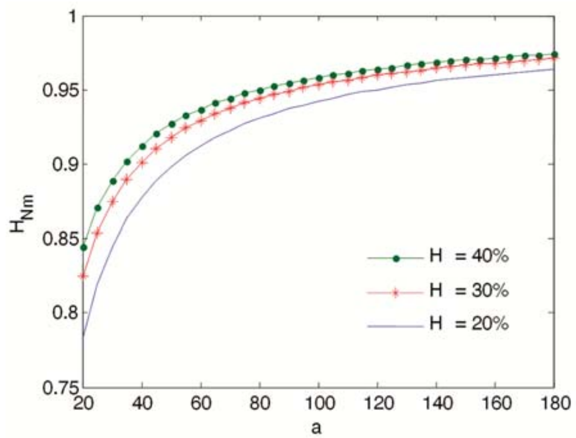

Fig. 12 — Effect of hematocrit on mean hematocrit for fixed values of $\tau_d = 0.2, \mu_p = 1.2cp, \mu_c = 4.0cp, \lambda_1 = 0.1$

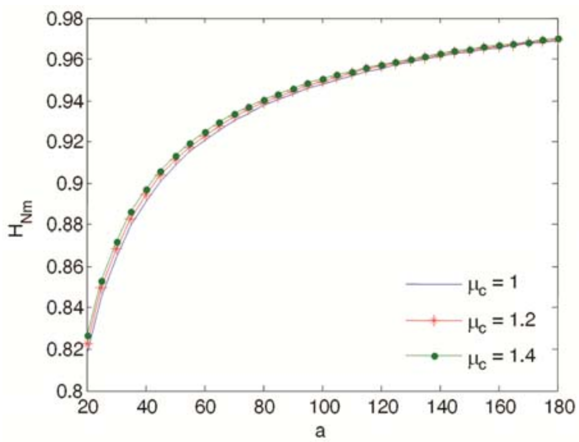

Fig. 13 — Effect of Casson fluid viscosity on mean hematocrit for fixed values of $H = 40\%, \tau_d = 0.2, \mu_p = 1.2cp, \lambda_1 = 0.1$

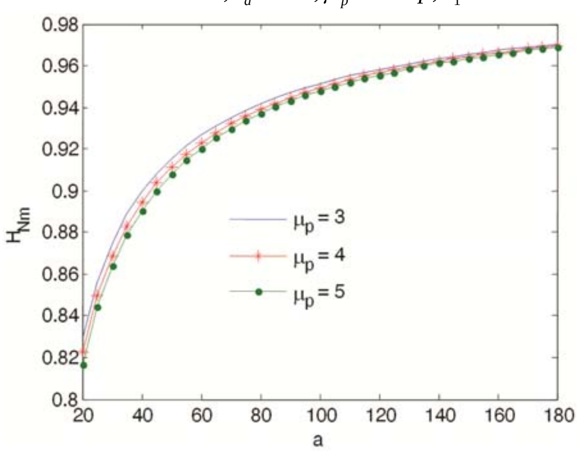

Fig. 14 — Effect of Newtonian fluid viscosity on mean hematocrit for fixed values of $H = 40\%, \tau_d = 0.2, \mu_c = 4.0cp, \lambda_1 = 0.1$

Figures (7) - (11) represent the influence of various physical parameters on central hematocrit. Central hematocrit increases with an increase in yield stress, which is shown in figure (7). From figure (8) it is

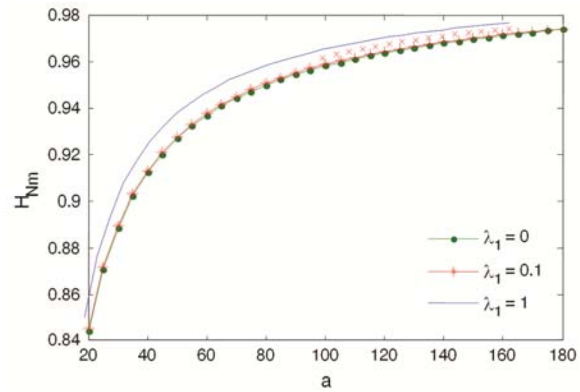

Fig. 15 — Effect of Jeffery parameter on mean hematocrit for fixed values of $H = 40\%, \tau_d = 0.2, \mu_c = 4.0cp, \mu_p = 1.2cp$

noticed that as the hematocrit increases the central hematocrit decreases. The important point to be noticed is the impact of central viscosity and outer viscosity. The important point to be noticed is the impact of central viscosity and outer viscosity. From figure (9&10) central hematocrit increases with the increase of Casson viscosity and Jeffrey parameter respectively.

Mean hematocrit was observed by changing different physical parameters through figures (12) – (16). Mean hematocrit increases with an increase in yield stress and hematocrit which can be observed from the figures (12) & (13). It is depicted from the figure (14) that as central viscosity increases the mean hematocrit is increasing. As outer viscosity is increasing the mean hematocrit is decreasing as observed by the figure (15). Figure (16) says that as the Jeffery parameter increases the mean hematocrit increases.

5 Conclusions

This paper accepts the results of [27] if $\lambda_1 \to 0$ *and* $\tau_d \to 0$. Erythrocytosis is one of the diseases that can be affected by a person due to an increase in hematocrit. Hematocrit increases due to dehydration or splenic contraction which occurs when a person is excited or over-exercised. So changes in central hematocrit, mean hematocrit and effective is observed in the present paper by varying many physical parameters and the following conclusions were found.

1. Effective viscosity, central hematocrit and mean hematocrit increase with an increase in yield stress.
2. As hematocrit increases, there is an increase in effective viscosity and mean hematocrit but a decrease in central hematocrit.

3. Increment in central viscosity finds increment in Effective viscosity, central hematocrit and mean hematocrit.
4. Central and mean hematocrit decreases with an increase in outer viscosity, but effective viscosity increases with an increase in outer viscosity.
5. Jeffery parameter finds Effective viscosity, central hematocrit and means hematocrit is more by its presence.

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Nomenclature

u	Velocity
τ_0	Yield stress
μ	Consistency factor
r	Radius
z	Axis
a	Radius of the peripheral region
b	Radius of the core region
r_0	Radius of the plug region
H_c	Core hematocrit
H	Hematocrit of blood leaving or entering the tube
λ_1	Jeffrey parameter
η	Peristaltic wave
c	Wave speed
H_{Nc}	Normalized core hematocrit
H_m	Mean hematocrit
H_{Nm}	Normalized mean hematocrit
μ_c	Viscosity of core fluid
μ_p	Viscosity of peripheral region
F	Flux of the fluid
F_c	Flux in core region
F_p	Flux in peripheral region

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