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Quantum Electron Acoustic Solitons and Double Layers with κ-deformed Kaniadakis Distributed Electrons

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The present investigation is dedicated to the study of propagation characteristics of electron acoustic (EA) waves and double layers in the quantum plasma system containing inertialess hot electrons and inertial cold electrons with stationary ions forming the charge neutralizing background. It is assumed that hot electrons follow κ-deformed Kaniadakis distribution as governed by the parameter κ. Using the appropriate stretched coordinates and reductive perturbation method (RPM) the Korteweg-de Vries (KdV) and modified KdV (mKdV) equations have been derived. For the sake of analysis, a limit of range of deformation parameter (κ) has been set as $-0.4 \leq \kappa \leq 0.4$. For the defined range, it has been observed that plasma system supports rarefactive solitary structures. The amplitude and width of KdV soliton have been significantly affected by the quantum effects and remains unaffected by the deformation parameter (κ). The analysis was further extended to the derivation of mKdV equation to investigate the existence of small amplitude double layers (DLs). Only negative potential DLs are found to exist whose dynamics significantly depends on deformation parameter (κ), quantum effects (H) and hot to cold electron density ratio (α). The outcome of the present discourse may be helpful to understand the use of generalized entropies in the environment of plasma physics.

Keywords: Electron acoustic waves, κ-deformed Kaniadakis distribution, Reductive Perturbation Method, Korteweg-de Vries equation, Modified Korteweg-de Vries equation, Double layers.

1 Introduction

The exploration of various features of quantum plasma have received immense assiduity in last few years by virtue of its far reaching applications in high density astrophysical environments such as white dwarfs, neutron stars etc[1,2]. The signatures of quantum plasma are encountered not only in laser produced plasmas[3], semiconductor devices[4,5], quantum dots and nanowires[6], quantum wells and diodes[7,8] but also in biophotonics [9] and cool vibes[10]. It may be mentioned that the study of quantum plasma becomes significant when de-Broglie wavelength associated with the charged particles becomes comparable to or greater than interparticle distance of the system and plasma acts like Fermi gas and hence quantum mechanical effects become crucial[11-13]. To describe the dynamics of quantum-scale plasma particles, quatum hydrodynamic (QHD) model is certified which is the generalization of the classical fluid model with the inclusion of a correction term pronounced as "Bohm Potential", which subsequently describes the new aspects of collective interactions at the nanoscale[14]. Quantum effects play crucial role in the various features of nonlinear quantum wave structures as reported by a several authors[13,15-20]. The electron acoustic (EA) wave is a small amplitude wave that occurs in plasma with two distinct populations of electrons, referred as “cool” and “hot”[21] and its presence can be witnessed in the electron-ion plasmas containing ions hotter than electrons. EA wave is actually an electrostatic wave within which the cool electrons provide the momentum and the restoring force comes only from the hot electrons[22]. The ions play the part of the neutralizing background, i.e. their dynamics do not affect the EA waves since the frequency of the EA wave is much greater than the frequency of plasma ion. In order to avoid damping of the wave, the phase speed of the EA wave should be intermediate between the thermal hot and cold electron speeds[23]. The EA waves occur in the laboratory as well as in space plasmas, for example, in the bow shock of the Earth, within auroral magnetosphere[24] and in a geomagnetic tail.

Various particle distributions enhance the bountifulness of variety of the wave motions in plasma through their significant contribution in describing the physics of nonlinear wave structures.

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Also, they considerably influence the requisite conditions for the creation of solitons and double layers. Several observations illustrate the inability of Maxwellian distribution of particles to describe the non-equilibrium attribute of the plasma. For such considerations, non-Maxwellian distribution functions present good approximations. In 2001, Kaniadakis[25] proposed a new one parameter deformation of exponential function known as κ-deformed distribution and proved that it could accommodate the traditional Maxwell-Boltzmann as well as the non-extensive distribution. He further suggested that such a κ-deformed distribution could be seen as the result of a more simplified statistics known as superstatistics. The Kaniadakis non-Gussian statistics is elaborated by κ-entropy which occurs naturally in the infrastructure of so-called kinetic interaction theory, which describes nonlinear kinetics in the particle systems[26]. In the framework of this principle, the expression of Fokker–Planck equation depicting the kinetic evolution of system gets modified by imposing the generalized entropy associated with the system. The fabulous similitude of structures of κ-deformed statistics with that of special relativity reflects its relevance self consistent formulation of relativistic statistical theory[25,27] and relativistic Boltzmann kinametics[28]. The various applications of κ-deformed emanating from Kaniadakis entropy include the relativistic flux distribution of cosmic rays[27], formation of quark-gluon plasma[26], nonlinear kinetics[29], kinetics of interacting atoms and photons[30]. Other relativistic contexts such as nuclear kinetics[31], gas in an electromagnetic field[32] and wave particle interactions[33] also find an important role of κ-deformed Kaniadakis distribution. Lourek and Tribeche[34] employed the Sagdeev approach to investigate the features of ion-acoustic (IA) solitary waves and double layers in unmagnetized electron-ion plasma using κ-deformed Kaniadakis distribution. They observed that κ-deformed parameter slightly changes the IA structures. Ourabah and Tribeche[35] and Ourabah *et al*[36] explored the blackbody radiation and quantum entanglement within the background of κ-deformed distribution arising out of Kaniadakis entropy. Gougam and Tribeche[37] examined the effects of this distribution on small amplitude EA double layers. Saha and Tamang[38] investigated positron-acoustic waves in four component plasma consisting of static positive ions, mobile cold positrons and Kaniadakis distributed hot positrons and electrons. They observed that κ-deformed parameter has no effect on the solitary wave solution of KdV equation whereas it influences the solution of mKdV equation. Abul-Magd *et al*[39] proposed the non-Gaussian deformations (κ≠0) of the conventional orthogonal and unitary ensembles of random matrices. Khalid *et al*[40] used the κ-deformed Kaniadakis distribution of electrons in the context of arbitrary amplitude IA solitary waves in two fluid magnetized plasma. They discussed the effect of various parameters such as Mach number, strength of magnetic field and obliqueness on the soliton dynamics. In order to join hands with the flourishing studies engrossing the generalized entropies in nonlinear waves, we give consideration to the rational use of κ-deformed Kaniadakis entropy. In the present investigation, we intend to study the nonlinear EA waves in quantum plasma with κ-deformed Kaniadakis distributed electrons. The main aim of this work is to check the quantum effects in the framework of κ-deformed Kaniadakis entropy on the physical structures and existence conditions of EA waves and double layers. To illustrate this work, we will derive the Korteweg-de-Vries (KdV) and modified KdV (mKdV) equations by using reductive perturbation technique in sections 3 and 4 respectively. The analysis is further extended to derive the solution of small amplitude double layers. A detailed discussion of numerical results is presented in section 5 and finally the conclusion is given in section 6.

2 Basic Equations and Dispersion Relation

The nonlinear characteristics of the propagation of EA waves in three component unmagnetized quantum plasma system containing inertialess hot electrons, inertial cold electrons and stationary ions is considered here. The stationary ions provide the charge neutralizing background and the hot electrons are assumed obey by κ-deformed Kaniadakis distribution. Further, the wave phase velocity lie between thermal velocities of hot and cold electrons and ion plasma frequency is assumed to be much smaller than wave frequency. For such a plasma system, the normalized equations executing the dynamics of EA waves are given as

$$\frac{\partial n_c}{\partial t}+\frac{\partial (n_c v_c)}{\partial x}=0 \qquad ...(1)$$

$$\frac{\partial v_c}{\partial t}+v_c\frac{\partial v_c}{\partial x}=\alpha\frac{\partial \phi}{\partial x}-2\alpha n_c\frac{\partial n_c}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_c}}\frac{\partial^2 n_c}{\partial x^2}\right) \qquad ...(2)$$

$$\frac{\partial^2\phi}{\partial x^2} = n_h + \frac{1}{\alpha} n_c - \mu_i \quad ...(3)$$

where the various quantities are normalized as follows: the number densities of cold and hot electrons n_c and n_h by their equilibrium densities, the cold electron velocity v_c by $C_e = \sqrt{E_f / \alpha m_e}$, the value of electrostatic wave potential ϕ by E_f / e, the space (x) and time coordinates by Debye length of electron (λ_D) and inverse of the plasma frequency of cold electron ω_{pc}^{-1}. Here $\omega_{pc}^{-1} = (\varepsilon_0 m_e / n_{c0} e^2)^{1/2}$, $\lambda_D = C_e / \omega_{pc}$, $E_f = \hbar^2 (3\pi^2 n_{c0})^{2/3} / 2m_e$, $\alpha = n_{h0} / n_{c0} > 1$, m_e is the mass of the electron and magnitude of electron charge is 'e', The nondimensional quantum parameter H measures the effects of quantum diffraction and is defined as $H = \hbar\omega_{pc} / m_e C_e^2$ where $\hbar = h / 2\pi$ and 'h'represents the Planck's constant. To explain the motion of electrons, the κ-deformed Kaniadakis distribution is followed. A detailed description of this distribution is presented by Kaniadakis[25], Gougam and Tribeche[37] and Khalid *et al*[40] in their studies. The statistics of Kaniadakis are based on a κ-deformed exponential[25] defined as

$$\exp_\kappa(x) = \left(\sqrt{1+\kappa^2 x^2} + x\right)^{1/\kappa}$$

where κ is the strength of deformation and the acceptable value of κ must satisfy $|\kappa| \le 0.4$. It is mentioned that the function $\exp_\kappa(x)$ reduces to the standard exponential in the κ→0 limit, i.e. it has property $\exp_0(x) = \exp(x)$. Its behavior is similar to the standard exponential for $x \to 0$. It is interesting that the Taylor expansion's first three terms are the same as the standard exponential:

$$\exp_\kappa(x) = 1 + x + \frac{x^2}{2} + \left(1-\kappa^2\right)\frac{x^3}{6} + ...$$

Further, the normalized number density of hot electron with κ-deformed Kaniadakis distribution is written as:

$$n_h = \left(\sqrt{1+\kappa^2 x^2} + x\right)^{1/\kappa} \quad ...(3)$$

To linearize the set of basic equations (1)-(3), we take the following perturbation expansions of various field quantities about their equilibrium value: $n_c = 1 + n_c^{(1)}$, $v_c = v_c^{(1)}$, $\phi = \phi_1$ and $n_h = 1 + \phi_1$. Now it is assumed that these field variable vary as the $\exp(\iota kx - \iota\omega t)$ and we get linear dispersion relation:

$$\omega^2 = k^2\left(2\alpha + \frac{1}{1+k^2}\right) \quad ...(4)$$

From this relation it is clear that, the wave frequency (ω) is the function of propagation constant (k) and density ratio of hot to cool electrons (α). Therefore, the wave velocity is a function of these parameters. It also becomes clear from equation (4) that ω is independent of deformation parameter (κ). It is further mentioned that the wave frequency (ω) relies on quantum parameter (H) via parameter α, where the quantum parameter H is calculated as

$$H = \frac{2e\alpha}{\hbar}\sqrt{\frac{m_e}{\varepsilon_0}}\left(\frac{1}{3\pi^2 n_{c0}^{1/4}}\right)^{2/3} \quad ...(5)$$

The above equation gives the mathematical relation of quantum parameter (H) with different parameters such as mass of electron (m_e), electron charge (e), cold electron density (n_{c0}) and α.

3 Korteweg-de Vries (KdV) Equation and its Solution

A nonlinear theory is established regarding EA waves to investigate solitary structures in the given quantum plasma system by using reductive perturbation technique. This will result in scaling of independent variables by stretched coordinates in the space (X) and time (T) as: $X = \varepsilon^{1/2}(x - \lambda t)$, $T = \varepsilon^{3/2} t$. Here ε is a small parameter and λ is phase velocity of the wave. The expansion of perturbed quantities n_c, v_c and ϕ about their equilibrium values are given as:

$$\begin{aligned} n_c &= 1 + \varepsilon n_c^{(1)} + \varepsilon^2 n_c^{(2)} + \varepsilon^3 n_c^{(3)} + ... \\ v_c &= \varepsilon u_c^{(1)} + \varepsilon^2 u_c^{(2)} + \varepsilon^3 u_c^{(3)} + ... \\ \phi &= \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + ... \end{aligned} \quad ...(6)$$

and n_h is distributed by κ-deformed Kaniadakis distribution is given by

$$n_h = 1 + \varepsilon\phi_1 + \varepsilon^2\left(\phi_2 + \frac{\phi_1^2}{2}\right) + \varepsilon^3\left(\phi_3 + \phi_1\phi_2 + \left(\frac{1-\kappa^2}{6}\right)\phi_1^3\right) + ... \quad ...(7)$$

Using the expansion (6) of the perturbed quantities into equations (1) to (3) and picking the lowest order terms of ε we get the phase velocity and first order quantities as $\lambda = \sqrt{1+2\alpha}$, $n_c^{(1)} = -\alpha\phi_1$, $u_c^{(1)} = -\alpha\lambda\phi_1$. Now by picking the next higher order terms of ε we get

$$-\lambda\frac{\partial n_c^{(2)}}{\partial X} + \frac{\partial n_c^{(1)}}{\partial T} + \frac{\partial u_c^{(2)}}{\partial X} + \frac{\partial\left(n_c^{(1)}u_c^{(1)}\right)}{\partial X} = 0 \qquad ...(8)$$

$$-\lambda\frac{\partial u_c^{(2)}}{\partial X} + \frac{\partial u_c^{(1)}}{\partial T} + u_c^{(1)}\frac{\partial u_c^{(1)}}{\partial X} = \alpha\frac{\partial \phi_2}{\partial X} - 2\alpha\frac{\partial n_c^{(2)}}{\partial X} - 2\alpha n_c^{(1)}\frac{\partial n_c^{(1)}}{\partial X} + \frac{H^2}{4}\frac{\partial^3 n_c^{(1)}}{\partial X^3} \qquad ...(9)$$

$$\frac{\partial^2\phi_1}{\partial X^2} = \phi_2 + \frac{1}{2}\phi_1^2 + \frac{1}{\alpha}n_c^{(2)} \qquad ...(10)$$

Eliminating $n_c^{(2)}$, $n_c^{(1)}$, $u_c^{(2)}$ $u_c^{(1)}$ and ϕ_2 from equations (8) to (10), we get the required KdV equation after replacing $\phi_1 = \phi$ as given below

$$\frac{\partial\phi}{\partial T} + A\phi\frac{\partial\phi}{\partial X} + B\frac{\partial^3\phi}{\partial X^3} = 0 \qquad ...(11)$$

Where, $A = -\frac{1}{2\sqrt{1+2\alpha}}\left(1+3\alpha+8\alpha^2\right)$ and

$$B = \frac{1}{2\sqrt{1+2\alpha}}\left(1-\frac{H^2}{4}\right) \qquad ...(12)$$

Here A and B respectively represents the nonlinearity and dispersion coefficients. To find the stationary solution of equation (11), we transform the independent variables X and T into one variable $\eta = X - MT$, where M is the constant velocity of the solitary wave. By applying the appropriate boundary conditions such as $\frac{\partial^2\phi}{\partial\eta^2} \to 0$, $\frac{\partial\phi}{\partial\eta} \to 0$ and $\phi \to 0$ at $\eta = \pm\infty$, the stationary solution of equation (11) becomes

$$\phi_s = \phi_0 \sec h^2\left(\frac{\eta}{W}\right) \qquad ...(13)$$

Here $\phi_0 = 3M/A$ represents the amplitude and $W_s = \sqrt{4B/M}$ represents the width of the soliton.

4. Modified Korteweg-de Vries (mKdV) Equation and Double Layer Solution

In order to analyze the nonlinear propagation of EA waves and to derive the modified KdV (mKdV) equation in quantum plasma, we need some equations containing higher order coefficients to interpret such system properly. To derive mKdV equation, the stretched coordinates used are $X = \varepsilon(x - \lambda t)$, $T = \varepsilon^3 t$. Using equations (6) and (7) in (1)-(3) and collecting the terms of different powers of ε, we obtain following second order quantities

$$n_c^{(2)} = -\alpha\phi_2 - \frac{\alpha}{2}\phi_1^2,$$

$$u_c^{(2)} = -\lambda\alpha\phi_2 - \frac{\lambda\alpha}{2}\phi_1^2 - \lambda\alpha^2\phi_1^2 \qquad ...(14)$$

For higher order of ε we will get

$$-\lambda\frac{\partial n_c^{(3)}}{\partial X} + \frac{\partial n_c^{(1)}}{\partial T} + \frac{\partial u_c^{(3)}}{\partial X} + \frac{\partial\left(n_c^{(1)}u_c^{(2)}\right)}{\partial X} + \frac{\partial\left(n_c^{(2)}u_c^{(1)}\right)}{\partial X} = 0 \qquad ...(15)$$

$$-\lambda\frac{\partial u_c^{(3)}}{\partial X} + \frac{\partial u_c^{(1)}}{\partial T} + u_c^{(1)}\frac{\partial u_c^{(2)}}{\partial X} + u_c^{(2)}\frac{\partial u_c^{(1)}}{\partial X} = \alpha\frac{\partial\phi_3}{\partial X} - 2\alpha\frac{\partial n_c^{(3)}}{\partial X} - 2\alpha n_c^{(1)}\frac{\partial n_c^{(2)}}{\partial X} - 2\alpha n_c^{(2)}\frac{\partial n_c^{(1)}}{\partial X} + \frac{H^2}{4}\frac{\partial^3 n_c^{(1)}}{\partial X^3} \qquad ...(16)$$

$$\frac{\partial^2\phi_1}{\partial X^2} = \phi_3 + \phi_1\phi_2 + \frac{\left(1-\kappa^2\right)}{6}\phi_1^3 + \frac{1}{\alpha}n_c^{(3)} \qquad ...(17)$$

Now, using equations (14)-(17) we get required mKdV equation as

$$\frac{\partial\phi_1}{\partial T} + A\frac{\partial\left(\phi_1\phi_2\right)}{\partial X} + B\frac{\partial^3\phi_1}{\partial X^3} + S\phi_1^2\frac{\partial\phi_1}{\partial X} = 0 \qquad ...(18)$$

where A and B are given by equation (12) and S is cubic nonlinearity coefficient given by

$$S = -\frac{3}{2\sqrt{1+2\alpha}}\left(\frac{\left(1-\kappa^2\right)}{6} + \frac{3\alpha}{2} + 6\alpha^2 + 4\alpha^3\right) \qquad ...(19)$$

It is mentioned that the mKdV solitons exist only for positive values of S, i.e. S > 0. We have calculated numerically that for -0.4 ≤ κ ≤ 0.4 and $\alpha > 1$, the cubic nonlinearity coefficient S is negative thereby indicates that mKdV solitons doesn't exist in the present study. Furthermore, relative to the solitons, the double layer (DL) is a higher nonlinear structure. While interacting with such structure in the nonlinear progression equation, one is to preserve the term A as well. Here, we introduce the DL solution corresponding to the mKdV equation in this section and discuss the circumstances under which double layers can be created. When A→0 but A≠0 then equation (18) reduces to the form as given below

$$\frac{\partial \phi_1}{\partial T} + D\phi_1 \frac{\partial \phi_1}{\partial X} + B\frac{\partial^3 \phi_1}{\partial X^3} + S\phi_1^2 \frac{\partial \phi_1}{\partial X} = 0 \quad ...(20)$$

On using[41] $A\phi_2 \to D\phi_1/2$, equation (20) becomes the required mKdV equation. In order to acquire the solution the independent variables X and T are transformed to single variable $\eta = X - MT$. Therefore equation (20) is transformed to

$$\frac{1}{2}\left(\frac{d\phi_1}{d\eta}\right)^2 + V(\phi_1) = 0 \quad ...(21)$$

where V(ϕ_1) is the Sagdeev potential and is given by:

$$V(\phi_1) = a_2\phi_1^2 + a_3\phi_1^3 + a_4\phi_1^4 \quad ...(22)$$

Here $a_1 = -M/2B$, $a_2 = D/6B$ and $a_3 = S/12B$ and we have used the boundary conditions as $\frac{d\phi_1}{d\eta} \to 0$, $\phi_1 \to 0$ at $|\eta| \to \infty$. Sagdeev potential should have been negative between $\phi_1 = 0$ and $\phi_{dl} = 0$ for the creation of double layers, ϕ_{dl} being the amplitude of DLs must fulfil the conditions: $V(\phi_1) = 0$ at $\phi_1 = 0$ and $\phi_{dl} = 0$, $V'(\phi_1) = 0$ at $\phi_1 = 0$ and $\phi_{dl} = 0$ and $V''(\phi_1) = 0$ at $\phi_1 = 0$ and $\phi_{dl} = 0$. Replacing $V(\phi_1)$ by $V(\phi)$ for the convenience, we get

$$V(\phi) = a_4\phi^2(\phi_{dl} - \phi)^2 \quad ...(23)$$

The double layer solution is

$$\phi_{DL} = \frac{\phi_{dl}}{2}\left[1 - \tanh\left(\frac{2}{W}\eta\right)\right] \quad ...(24)$$

and width and amplitude of DLs is given by

$$W_{dl} = \frac{1}{|\phi_{dl}|}\sqrt{\frac{-8}{a_4}} \text{ and } \phi_{dl} = -\frac{a_3}{a_4} \quad ...(25)$$

The above relations for the amplitude and width of DLs equation clearly describe that with the increase in the values of amplitude of the wave, width and velocity of the DLs change.

5 Results and Discussion

In this paper, the different characteristics of EA waves in quantum plasma have been discussed in the environment of κ-deformed Kaniadakis distributed hot electrons. In previous sections, derivations of KdV and mKdV equations with the double layers solutions have been discussed. This section is devoted to the numerical analysis of the effects of various parameters such as hot to cold electron number density (α), quantum parameter (H) and deformation parameter (κ) on the soliton and DL dynamics. As mentioned earlier, the dispersion relation (4) indicates that the wave frequency (ω) is independent of deformation parameter (κ) and is a function of quantum parameter (*H*) via parameter α (equation (5)). Fig. 1 represents a plot drawn between wave frequency (ω) and wave number (*k*) for three different values of hot to cold electron density ratio (α). As α is always greater than 1, so values of α are taken as 2.0, 3.0 and 4.0 and are shown by solid, dotted and dashed lines respectively. A linear relationship between ω and k is observed and the wave frequency is found to increase with the rise in hot to cold electron density ratio (α). It is further revealed that the value of quantum parameter (H) varies with change in α. Hence, quantum effects also influence the wave frequency as depicted in Fig. 2. A similar kind of disposition was observed by Chandra *et al*[42] in their research. Fig.2 displays a plot of wave frequency (ω) and quantum parameter (*H*) at three different values of cold electron density (n_{c0}) as shown respectively by solid ($n_{c0} = 1.2\times10^{33} m^{-3}$), dotted ($n_{c0} = 1.5\times10^{33} m^{-3}$) and dashed lines ($n_{c0} = 1.7\times10^{33} m^{-3}$). It is observed that the wave frequency (ω) increases with the increase in quantum effects (*H*) as well as cold electron density (n_{c0}).

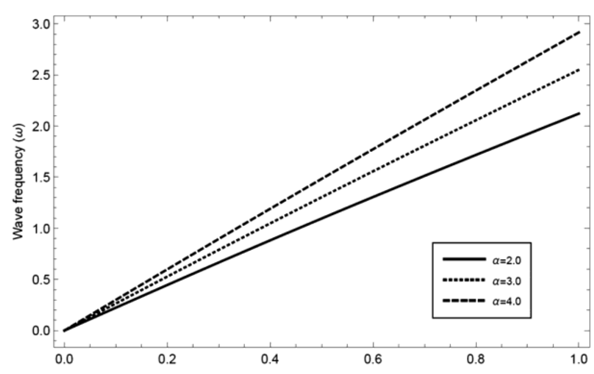

Fig. 1 — Plot of dispersion relation at three different values of α.

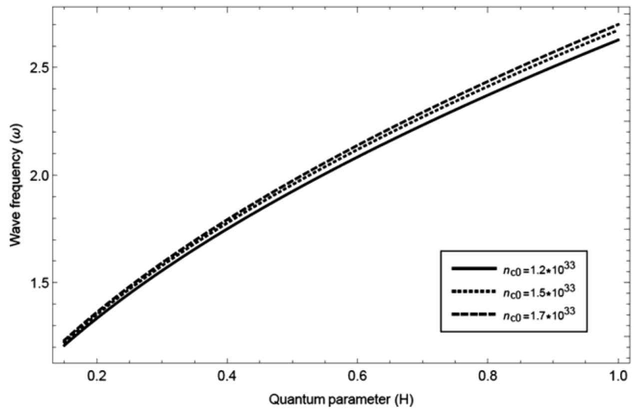

Fig. 2 — Wave function (w) as a function of quantum parameter (H) at different values of n_{c0}.

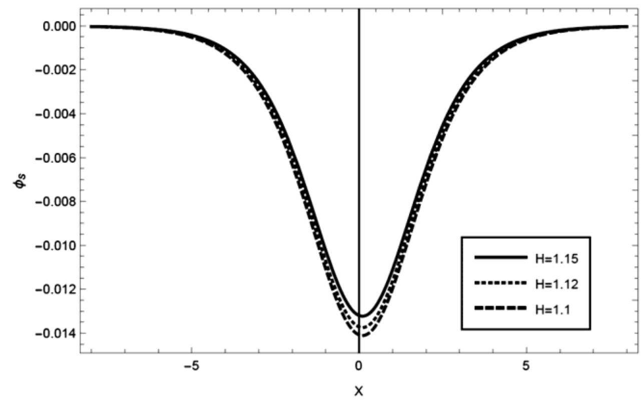

Fig. 3 — Variation of KdV solitons potential ϕ_s with respect to space coordinate (X) for three different values of H with $n_{c0} = 1.7\times10^{33}$ and M=0.1.

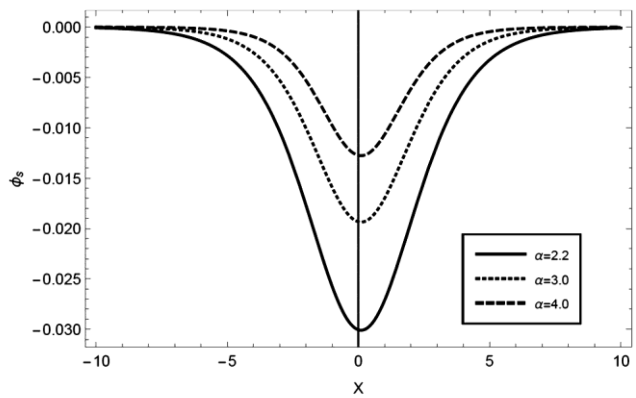

Fig. 4 — KdV solitons potential ϕ_s as a function of space coordinate (X) at different density ratios α with M=0.1.

It is worth mentioning that the nature and magnitude of the nonlinear coefficient A and dispersion coefficient B determine the characteristics of the soliton structures. From equation (12), we see that coefficient A depends on α only, whereas B depends upon both density ratio (α) and quantum parameter H. As we know the coefficient H is a function of α, hence the nonlinear coefficient A is also influenced by this factor. Solving equation (11) for B=0, we get H=2.0. Hence for existence of soliton solution given by equation (13), M must be positive for the range $0 \le H \le 2$ whereas for the range H>2, the soliton should have negative velocity (M). Hence these conditions of quantum parameter and the sign of coefficient A determine the occurrence of rarefactive ($\phi_0<0$) or compressive ($\phi_0>0$) soliton. However, for H >2 and positive value of soliton velocity M, the square root term in the expression of B (equation (12)) becomes imaginary and one can assume periodic potential structures. It is pertinent to mention that in congruent with the earlier observations[37,42], only rarefactive solitons are reported in the present research problem. It is obvious from the expression of amplitude ϕ_0 and width W_s of solitary wave that these are independent of deformation parameter κ stating thereby that this parameter has no influence on the properties of KdV solitons. However, other parameters such as quantum effects and density ratio influence the soliton dynamics. Fig. 3 portrays the variation of rarefactive KdV soliton potential (ϕ_s) with respect to space coordinate (X) at three different values of quantum parameter. Here solid, dotted and dashed lines correspond respectively to H=1.15, H=1.12 and H=1.1 along with n_{c0}=1.7×10^{33}m^{-3} and M=0.1. It shows that with the increase in quantum effects, the amplitude of the solitons increases. To show the impact of electron number density ratio α on the soliton profile, Fig. 4 depicts the variation of ϕ_s vs X at three different values of α as indicated by solid (α=2.2), dotted (α=3) and dashed (α=4) lines. The other parameters are taken as n_{c0}=1.2×10^{33}m^{-3}, H=0.588 and M=0.1. It becomes clear from the plot that the amplitude of negative potential structures decreases with increase in electron density ratio α while the width gets decreased. It is pertinent to mention that a similar kind of behaviour has been observed by Zhu *et al*[43] in their research.

The analysis is further extended to explore the dynamics of modified KdV solitons using appropriate stretched coordinates and perturbation relation (6). In the process, mKdV equation (18) is derived, followed by the appearance of an additional term containing a cubic nonlinearity coefficient S. The value of S is found to be negative indicating thereby the non-existence of mKdV solitons in the present case. Hence, we preceded towards the exploration of small amplitude double layer (DL) dynamics by employing the mKdV equation (18) and obtained the double layer solution (24). In agreement with Gaugam and Tribeche[37] and Sahu[44] only negative potential double layers are obtained on which the effect of various parameters will be discussed. To investigate the impact of deformation parameter κ on the DL dynamics, Fig. 5 depicts the variation of DL-amplitude (ϕ_{dl}) of EA wave with quantum parameter H for the range of deformation parameter (κ) as $-0.4 \le \kappa \le 0$. Here solid line is for $\kappa = -0.4$, dotted line is for $\kappa = -0.3$ and dashed line is for $\kappa = 0.0$ with $n_{c0} = 1.2\times10^{33}$ and M=0.1. It is clear that, the amplitude of EA-DL increases with the increase in quantum parameter H. Further, at a fixed value of H, the DL-amplitude is found to increase with deformation parameter. The corresponding plot for the range $0 \le \kappa \le 0.4$ is shown in Fig. 6. It is observed that in this range, at a given value of H, the DL-amplitude decreases with the increase in deformation parameter κ as is clear from solid ($\kappa = 0.0$), dotted ($\kappa = 0.3$) and dashed ($\kappa = 0.4$) curves. The analogous plots for width are represented by Figs. 7 and 8 respectively which depict that the quantum effects decrease the DL-width. However at a particular value of H, for the deformation range $-0.4 \le \kappa \le 0$, the DL-width increases whereas it shows a decrease for the range $0 \le \kappa \le 0.4$ (see Figs. 7 and 8). In Fig. 9, we have plotted Sagdeev potential ($V(\phi)$) shown by equation (22) as a function of ϕ_{dl} for three different values of electron density ratio $\alpha = 2$ (solid line), $\alpha = 3$ (dotted line) and $\alpha = 3.5$ (dashed line) with M=0.1, $n_{c0} = 1.7\times10^{33} m^{-3}$ and $\kappa = 0.4$. It is evident that the peak amplitude of DLs increases with the increase in density ratio α. It worth mentioning that consistent scenario was observed by Sahu[44] in his research findings. In order to understand the effect of cold electron density (n_{c0}) on the DL profile, a plot of Sagdeev potential $V(\phi)$ as a function of ϕ_{dl} is shown in Fig. 10 at three values of $n_{c0} = 1.2\times10^{33} m^{-3}$ (solid line), $n_{c0} = 1.5\times10^{33} m^{-3}$ (dotted line) and $n_{c0} = 1.7\times10^{33} m^{-3}$ (dashed line) with M=0.1 and H=1.12 and κ=0.4. From the graph, it is obvious that the peak amplitude of EA-DLs increases with the increase in cold electron density n_{c0}. It is pertinent to mention that the deformation parameter (κ) has no effect on KdV soliton but it affects the DL dynamics significantly. A similar kind of behaviour has been observed by Saha and Tamang[38].

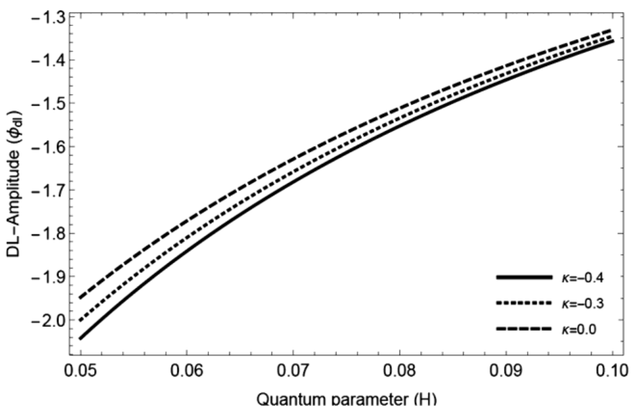

Fig. 5 — For the range -0.4≤κ≤0, plot of DL-amplitude (ϕ_{dl}) vs quantum parameter (H) for there different values of κ with M=0.1.

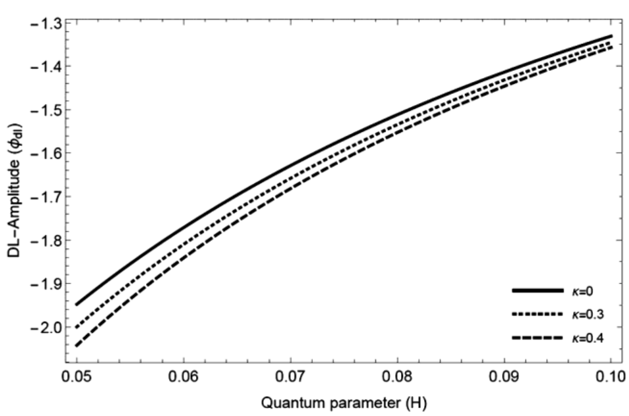

Fig. 6 — For the range -0.4≤κ≤0.4, plot of DL-amplitude (ϕ_{dl}) vs quantum parameter (H) for there different values of κ with M=0.1.

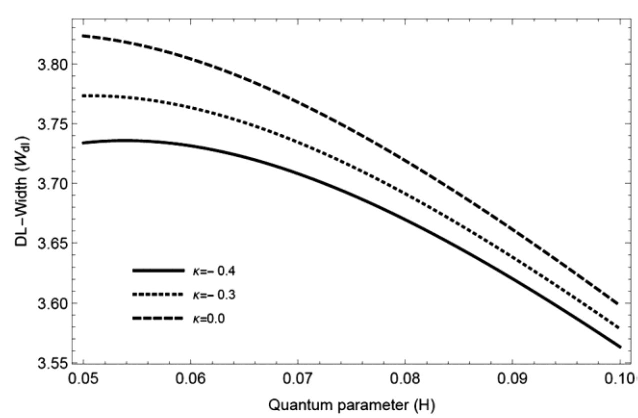

Fig.7 — For the range $-0.4 \le \kappa \le 0$, plot of DL-width (W_{dl}) vs quantum parameter (H) for there different values of κ with M=0.1.

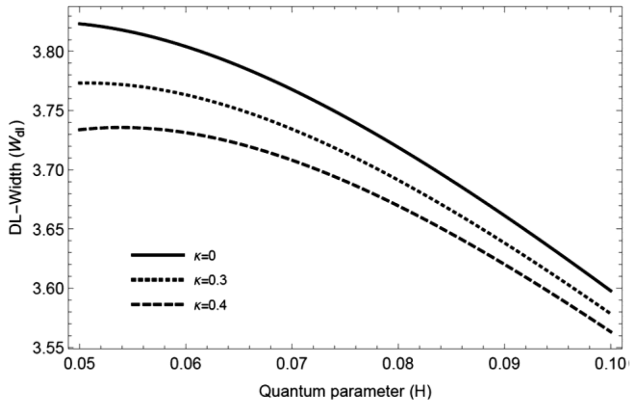

Fig. 8 — For the range -0.4≤κ≤0.4, plot of DL-width (W_{dl}) *vs* quantum parameter (*H*) for there different values of κ with *M*=0.1.

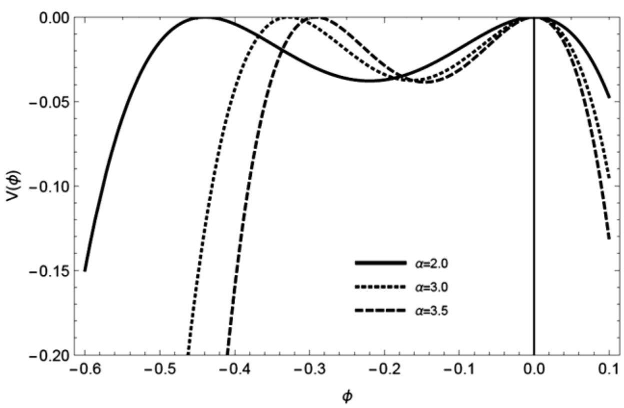

Fig. 9 — For DLs, plot of Sagdeev potential ($V(\phi)$) vs ϕ at three different values of α with n_{c0}=1.7×10^{33} with M=0.1 and κ =0.4.

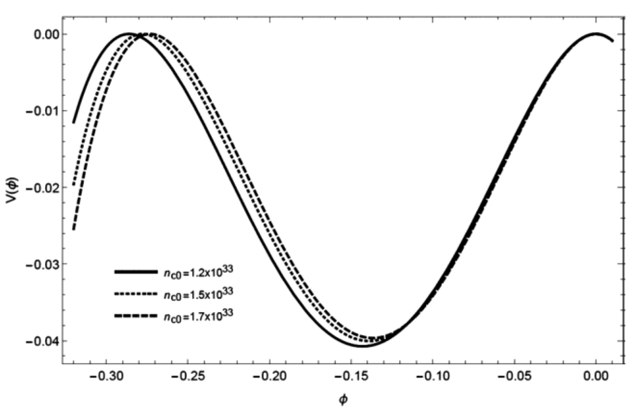

Fig. 10 — For DLs, plot of Sagdeev potential ($V(\phi)$) *vs* ϕ at three different values of n_{c0} with *H*=1.12, *M*=0.1 and κ =0.4.

6 Conclusion

The propagation characteristics of EA waves and double layers are analysed in quantum plasma system consisting of κ-deformed Kaniadakis distributed hot electrons. The wave frequency ω is a function of parameters such as electron density ratio α, quantum parameter *H* and cold electron density n_{c0}. The frequency is found to increase with α, H and n_{c0}. Using the conventional reductive perturbation method and appropriate stretched coordinates, nonlinear differential equation KdV equation is derived. Only negative potential solitary structures are observed in the present case for which the amplitude (width) increases (decreases) with quantum effects *H* and cold electron density n_{c0}. The study is further extended to explain the dynamics of DLs through the derivation of mKdV equation. Only negative potential EA-DLs are obtained that significantly depend on deformation parameter (κ), quantum parameter (H) and hot to cold electron density ratio (α). The DL-amplitude (width) increases (decreases) with quantum parameter H. However for the range of deformation parameter $-0.4 \le \kappa \le 0$ ($0 \le \kappa \le 0.4$), the DL-amplitude and width increase (decrease) with κ. DL-amplitude increases with both α and n_{c0}. The deformation parameter (κ) has no effect on KdV soliton but it affects the DL dynamics. The findings of the present investigation are consistent with those of Gaugam et al[37], Chandra et al[42], Sahu[43]and Saha and Tamang[44]. The present investigation may be helpful to understand the study of nonlinear waves in the laboratory and astrophysical situations.

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