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Flow of Dusty Non-Newtonian Fluid in a Vertical Channel in Presence of Inclined Magnetic Field

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A theoretical investigation of the influence of inclined magnetic field on the flow of non-Newtonian fluid with dust in a vertical channel has been considered. The non-Newtonian fluid is characterized by second-order fluid. The governing partial differential equations are obtained and solved analytically. The analytical expression for velocity field, temperature field , shearing stress flow flux at the channel are obtained. The potency of relevant parameters of the problem on velocity , shearing stress and flow flux of both dusty fluid and dust particles have been presented graphically.

Keywords: Second-order fluid, Dust particles, Shearing stress, Flow flux, Inclined magnetic field

1 Introduction

The non-linear relation between stress tensor and strain rate tensor has developed many non-Newtonian fluid models. The study of these fluids along with heat transfer is more valuable in fluid dynamics as they can explain the flow behavior of many industrial and artificial fluids such as lubricants, polymer solution, ketchup, custard *etc*. Second-order fluid is a sub-class of non-Newtonian fluid which can explain visco-elastic property of the fluid and also exhibit the normal stress effects. In nature most of the fluids exhibit the visco-elastic phenomena. Because of its characteristics and important applications in various industries some theoretical studies by many authors Nisa and Nazar[1], Abbas *et al.*[2], Hayat *et al.*[3], Rana *et al.*[4], Attia *et al.*[5], Hayat *et al.*[6], Luo *et al.*[7], Dey[8], Sreedevi[9], Sharma *et al.*[10] had explained the flow and heat transfer under different geometries.

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad \ldots (1)$$

The analysis of time dependent flow and heat transfer of a dusty visco-elastic fluid in presence of transverse of magnetic field has got many applications such as space heaters, power generators, nuclear reactor, batch settling, air craft icing and flow meters etc. Saffman[11] has discussed the stability of dusty gas in case of laminar flow. The flow and heat transfer of a dusty conducting fluid through a vertical channel in the presence of applied magnetic field has numerous applications such as MHD generators, flow meters , accelerators, and pumps. In these appliances, the solid particles in the form of dust are attached in the conducting fluid due to various physical and chemical activities. The presence of these particles on the operations of such devices has led to studies of flow behavior of conducting fluids.Various interesting cases on MHD oscillatory fluid flow with dust under different geometries has been analysed by Gupta *et al*[12]., Makinde *et al.*[13], Prakash *et al.*[14], Nayfeh[15], Singh *et al.*[16], Sivraj *et al.*[17], Khan *et al.*[18], Shanthi *et al.*[19], Bilal *et al.*[20], Prakash *et al.*[21].

Inspired by the above mentioned explorations and implementations, in this analysis influence of inclined magnetic field on dusty non-Newtonian fluid flow in a vertical channel is considered. The non-Newtonian fluid flow is characterizes by second-order fluid model.

The constitutive equation for the incompressible Second-order fluid is of the form
where σ is the stress tensor , A_n (n = 1,2) are the kinematic Rivlin-Ericksen tensors; μ_1, μ_2 and μ_3 are the material coefficients describing the viscosity, elasticity and cross-viscosity respectively. The material coefficients μ_1, μ_2 and μ_3 are taken constants with μ_1 and μ_3 as positive and μ_2 as negative Markovitz & Coleman[22] .The equation (1) was developed by Coleman & Noll[23].The perturbation method is used to solve the governing equations of the problem. The potency of relevant parameters of the problem on velocity, shearing stress and flow flux of both dusty fluid and dust particles have been presented

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graphically. The structure of the paper is consists of mathematical formulation, method of solution, results and discussion, conclusions and references.

2 Mathematical Formulation

Consider the unsteady couette flow of a conducting dusty non-Newtonian fluid in a vertical channel with radiative heat transfer. In Cartesian coordinate system, x-axis lies along the centre of the channel and y-axis is perpendicular to the channel walls. The distance between channel walls is a_1.The left and the right wall of the channel are represented by $\bar{y}=0$ and $\bar{y}=a_1$.The left wall of the channel is fixed and right wall is oscillating about a constant non-zero mean. An external uniform magnetic field of strength B_1 makes an angle θ with the positive direction of x-axis.

For investigating the governing fluid motion the following assumptions are considered:

1) The dust particles are assumed to be solid, evenly spread in the flow region electrically non-conducting.
2) The dust particles are spherical, equal in size and the number density N_0 is constant throughout the motion.
3) The intercommunication between the particles and chemical reaction have not been considered.
4) The magnetic Reynolds number is taken to be very small so that the induced magnetic field is negligible and the Hall effects have been neglected.
5) The channel is assumed to be infinite in length along the direction of $\bar{x}$ - axis so all physical properties are independent of $\bar{x}$ except pressure.

Under these suppositions (1-5), the equations that represent the motion of an electrically conducting non-Newtonian fluid through a vertical channel in presence of inclined magnetic field are given by

$$\frac{\partial\bar{u}}{\partial\bar{t}}=-\frac{1}{\rho}\frac{\partial\bar{P}}{\partial\bar{x}}+v_1\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+v_2\frac{\partial^3\bar{u}}{\partial\bar{t}\partial\bar{y}^2}-\frac{v_1}{k_1}\bar{u}+\frac{NK_1}{\rho}(\bar{u}_d-\bar{u})-\frac{\sigma_e B_1^2\sin^2\theta\,\bar{u}}{\rho}+g\beta_1(T-T_0) \quad \text{... (2)}$$

$$\frac{\partial\bar{u}_p}{\partial t}=K_1(\bar{u}-\bar{u}_p) \quad \text{... (3)}$$

$$\frac{\partial T}{\partial\bar{t}}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial\bar{y}^2}-\frac{1}{\rho c_p}\frac{\partial q_1}{\partial\bar{y}} \quad \text{... (4)}$$

The initial boundary conditions are:

$$\left.\begin{aligned}&\bar{u}(y,0)=\bar{u}_d(y,0)=0,\ T(y,0)=T_1,\\&\bar{u}(a_1,t)=\bar{u}_d(a_1,t)=U(1+\varepsilon e^{i\omega t}),\\&T(a,t)=T_2=T_0+(T_1-T_0)(1+e^{i\omega t}),\\&\bar{u}(0,t)=\bar{u}_d(0,t)=0,\ T(0,t)=T_0,\end{aligned}\right\} \quad \text{...(5)}$$

where u, u_d - velocities of fluid and dust particles in the x- direction, $\bar{t}$ - time, ω - frequency of oscillation, T , T_1 - fluid temperature and the initial fluid temperature, T_0, T_2 -the left and right wall temperature, N- the number density of dust particles, $\bar{P}$ - fluid pressure, g- acceleration due to gravity, q_1- radiative heat flux, β_1- coefficient of volume expansion,K_1- Stokes constant, c_p- specific heat at constant pressure, k- thermal conductivity, k_1- permeability porous medium ,θ-inclination of magnetic field, ε- small oscillation amplitude, σ_e - conductivity of the fluid, ρ - fluid density, $\nu_i=\frac{\mu_i}{\rho}$ where i=1,2.

The fluid is assumed to be optically thin with a relatively low density and the radiative heat flux is given by Cogley *et al.*[24]

$$\frac{\partial q_1}{\partial t}=4\alpha_1^{\,2}(T_0-T)$$

where α_1 is the mean radiation absorption constant.
we introduce the following dimensionless variables:

$$\bar{x}=\frac{x}{a_1},\ \bar{y}=\frac{y}{a_1},\bar{u}=\frac{u}{U_1},\theta_t=\frac{T-T_0}{T_1-T_0},\bar{t}=\frac{tU_1}{a_1},$$

$$Da=\frac{k_1}{a_1^{\,2}},M_1=\frac{\nu}{K_0a_1^{\,2}},l_1=\frac{N_0K_0a_1^{\,2}}{\rho\nu},\mathrm{Re}=\frac{U_1a_1}{\nu},$$

$$d=\frac{\nu_2U_1}{a_1\nu_1},\Pr=\frac{\nu\rho c_p}{k},N_1^{\,2}=\frac{4\alpha_1^{\,2}a_1^{\,2}}{k},$$

$$\bar{u}_d=\frac{u_d}{U_1},Gr=\frac{g\beta_1(T_1-T_0)a_1^{\,2}}{\nu U_1},\bar{P}=\frac{a_1P}{\nu\rho U_1},$$

$$H_1^{\,2}=\frac{a_1^{\,2}\sigma_eB_1^2}{\nu\rho},s_1^{\,2}=\frac{1}{Da}$$

where U_1 is the velocity of the mean flow.

The dimensionless governing equations together with the appropriate boundary conditions can be written as

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{\partial^2 u}{\partial y^2} + d\frac{\partial^3 u}{\partial t \partial y^2} -$$

$$(s_1^2 + H_1^2 \sin^2\theta + l_1)u + l_1 u_d + Gr\theta \qquad ...(7)$$

$$\mathrm{Re}\, M_1 \frac{\partial u_d}{\partial t} = u - u_d \qquad ... (8)$$

$$\mathrm{Re}\,\mathrm{Pr}\frac{\partial \theta_t}{\partial t} = \frac{\partial^2 \theta_t}{\partial y^2} + N_1^2\theta \qquad ...(9)$$

The corresponding boundary conditions are
$u(y, 0) = u_d(y, 0) = 0$, $\theta_t(y, 0) = 1$,
$u(1, t) = u_d(1, t) = 1+\varepsilon e^{i\omega t}$, … (10)
$\theta_t(1, t) = 1+\varepsilon e^{i\omega t}$,
$u(0, t) = u_d(0, t) = 0, \theta_t(0, t) = 0$.

where s_1 - porous medium shape factor parameter, Da - Darcy number, Gr- Grashof number, H_1- Magnetic parameter, l_1- particle concentration parameter, M_1- particle mass parameter, N_1- radiation parameter, Re- flow Reynolds number,d-visco-elastic parameter and Pr - Prandtl number.

3 Method of solution:

In order to solve the Eqs. (7), (8) and (9) for pure oscillatory flow, let

$$\left.\begin{aligned} &-\frac{\partial P}{\partial x} = \lambda_1 + \varepsilon e^{i\omega t}, u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t}, \\ &u_d(y,t) = u_{d0}(y) + \varepsilon u_{d1}(y)e^{i\omega t}, \\ &\theta_t(y,t) = \theta_0(y) + \varepsilon\theta_1(y)e^{i\omega t} \end{aligned}\right\} \qquad ...(11)$$

where λ_1- constant steady flow pressure gradient (Prakash *et al.*[21] & Saidu *et al.*[25])

Substituting the values from equation (11) in to equations (7)-(9), we obtain,

$$\frac{d^2u_0}{dy^2} - c_1^2 u_0 = -\lambda_1 - Gr\theta_0 \qquad ...(12)$$

$$c_4^2\frac{d^2u_1}{dy^2} - c_2^2 u_1 = -1 - Gr\theta_1 \qquad ... (13)$$

$$u_{d_0} = u_0 \qquad ...(14)$$

$$u_{d_1} = \frac{u_1}{1+\mathrm{Re}\, M_1 i\omega} \qquad ... (15)$$

$$\frac{d^2\theta_0}{dy^2} + N_1^2\theta_0 = 0 \qquad ... (16)$$

$$\frac{d^2\theta_1}{dy^2} + c_3^2\theta_1 = 0 \qquad ... (17)$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\text{at } y = 0 \\ &u_0 = u_{d_0} = u_1 = u_{d_1} = 0,\ \theta_0 = 0, \theta_1 = 0 \\ &\text{at } y = 1 \\ &u_0 = u_{d_0} = u_1 = u_{d_1} = 1,\ \theta_0 = 1, \theta_1 = 1 \end{aligned}\right\} \qquad ...(18)$$

where $c_1^2 = s_1^2 + 2l_1 + H_1^2\sin^2\theta$,

$c_2^2 = s_1^2 + l_1 + H_1^2\sin^2\theta - \dfrac{i}{1+\mathrm{Re}M_1 i\omega} + i\omega\mathrm{Re}$,

$c_3^2 = N_1^2 - i\omega\mathrm{Re}\,\mathrm{Pr}$, $c_4^2 = 1 + i\omega d$,

Velocity of dusty fluid(u) is given by

$$u = A_5 e^{c_1 y} + A_6 e^{-c_1 y} + \frac{\lambda_1}{c_1^2} + \frac{Gr\sin N_1 y}{(c_1^2 + N_1^2)\sin N_1} +$$

$$\varepsilon e^{i\omega t}\left(A_7 e^{c_5 y} + A_8 e^{-c_1 y} + \frac{1}{c_2^2} + \frac{Gr\sin c_3 y}{(c_3^2 c_4^2 + c_2^2)\sin c_3}\right)$$

velocity of dust particles(u_d) is given by

$$u_d = A_5 e^{c_1 y} + A_6 e^{-c_1 y} + \frac{\lambda_1}{c_1^2} + \frac{Gr\sin N_1 y}{(c_1^2 + N_1^2)\sin N_1} +$$

$$\frac{\varepsilon e^{i\omega t}}{(1+\mathrm{Re}iM_1\omega)}\left(A_7 e^{c_5 y} + A_8 e^{-c_1 y} + \frac{1}{c_2^2} + \frac{Gr\sin c_3 y}{(c_3^2 c_4^2 + c_2^2)\sin c_3}\right.$$

Skin Friction (τ_1) for the dusty fluid at y=a_1 is given by

$$\tau_1 = \frac{\partial^2 u}{\partial y^2} + d\frac{\partial^3 u}{\partial t\partial y^2}$$

and

Skin Friction (τ_2) for the dust particles at y=a_1 is given by

$$\tau_2 = \frac{\partial^2 u_d}{\partial y^2} + d\frac{\partial^3 u_d}{\partial t\partial y^2}$$

Flow flux of dusty fluid(f_1) is given by

$$f_1 = \int_0^1 u\,dy$$

$$= \frac{A_5}{c_1}(e^{c_1} - 1) - \frac{A_5}{c_1}(e^{-c_1} - 1) + \frac{\lambda_1}{c_1^2} -$$

$$\frac{Gr(\cos N_1 - 1)}{(c_1^2 + N_1^2)\sin N_1} + \varepsilon e^{i\omega t}\left(\frac{A_7}{c_5}(e^{c_5} - 1) - \frac{A_8}{c_5}(e^{-c_5} - 1) + \right.$$

$$\frac{1}{c_2^2} - \frac{Gr(\cos c_3 - 1)}{(c_3^2 c_4^2 + c_2^2)\sin c_3}$$

Flow flux of dust particles(f_2) is given by

$$f_2 = \int_0^1 u_d\,dy$$
$$= \frac{A_5}{c_1}(e^{c_1}-1) - \frac{A_5}{c_1}(e^{-c_1}-1) + \frac{\lambda_1}{c_1^2} - \frac{Gr(\cos N_1 - 1)}{(c_1^2+N_1^2)\sin N_1}$$
$$+ \frac{\varepsilon e^{i\omega t}}{1+\mathrm{Re} M_1 i\omega}\Big(\frac{A_7}{c_5}(e^{c_5}-1) - \frac{A_8}{c_5}(e^{-c_5}-1) + \frac{1}{c_2^2}$$
$$- \frac{Gr(\cos c_3 - 1)}{(c_3^2 c_4^2 + c_2^2)\sin c_3}$$

4 Results and Discussion:

To examine the influence of physical parameters on velocity, shearing stress and flow flux of dusty fluid and dust particles numerical results have been demonstrated graphically using MATLAB by assigning the following pertinent parameter values for computations unless otherwise indicated in the Fig. 1.

d=-.4, H_1=4, θ=π/3, Re=0.8, Pr=6, s_1=.4, M_1=0.4,
N_1=1, l_1=0.7, λ_1=0.2, Gr=5, t=1, ω=0.5, ε=0.001.

Figures 2-13, represent the velocity profile of both dusty fluid and particles to analyse the impact of various physical parameters.

Figures 2 & 3, display the effect of free convection parameter(Gr) on velocity profile. These figures notify that increase in Grashof number(Gr=5,6,7) leads to increase the buoyancy force in vertical channel consequently velocity of dusty fluid and dust particles also rises.

Figures 4 & 5, examine the influence of magnetic parameter(H_1) on velocity profile. The externally applied magnetic generates Lorentz force which opposes the motion of dusty fluid and dust particles . Hence velocity of both dusty fluid and dust particles follow a diminishing trend with improve in magnetic parameter (H_1=4,5,6).

Figures 6 & 7, suggest that with the rise in inclination angle(θ=π/6,π/3.π/2) of applied magnetic field(H_1) velocity of both fluid phase and dust phase decreases. Again, for θ=π/2 the maximum effort of Lorentz force is appeared in the Figs 6 & 7.

Figures 8 & 9, illustrate the effect of radiation parameter(N_1) on velocity profile. Due to increase radiation paparameter(N_1=1,1.5,2) the heat generation capacity of the flow system also increases thus dusty fluid and dust particles gets lighter. So, the velocity of both dusty fluid and dust particles gets accelerated with the rise in radiation parameter(N_1).

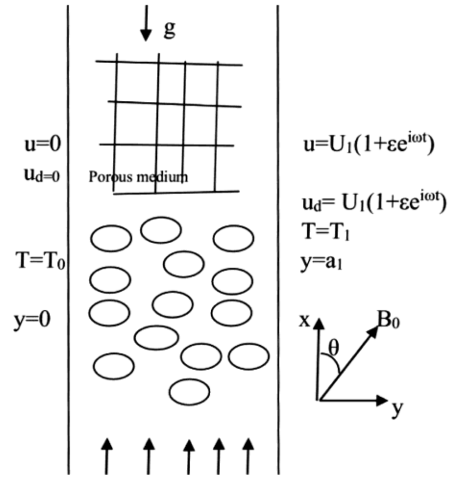

Fig. 1 — Physical model of the problem

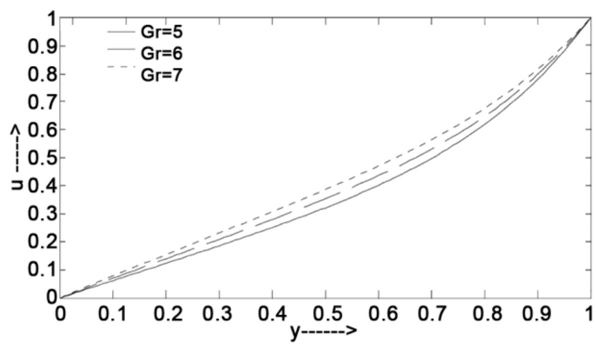

Fig. 2 — Effect of Gr on velocity profile u against y

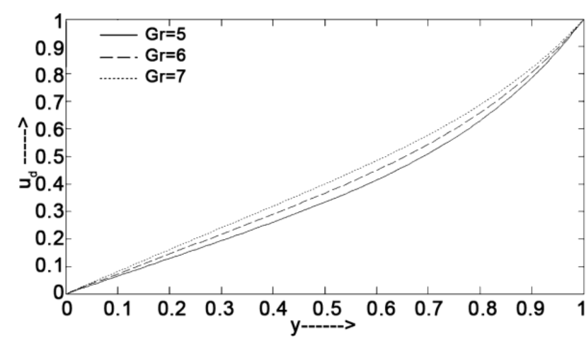

Fig. 3 — Effect of Gr on velocity profile u_d against y

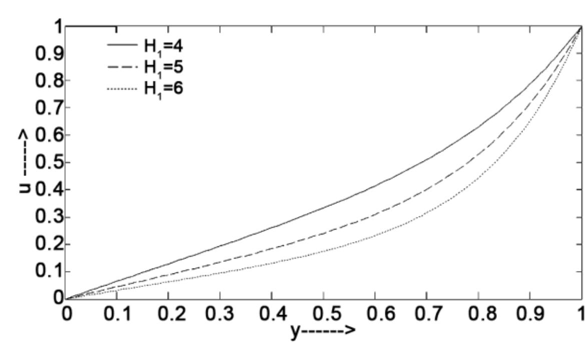

Fig. 4 — Effect of H_1 on velocity profile u against y

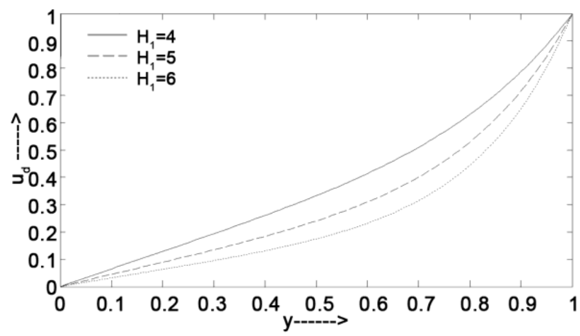

Fig. 5 — Effect of H_1 on velocity profile u_d against y

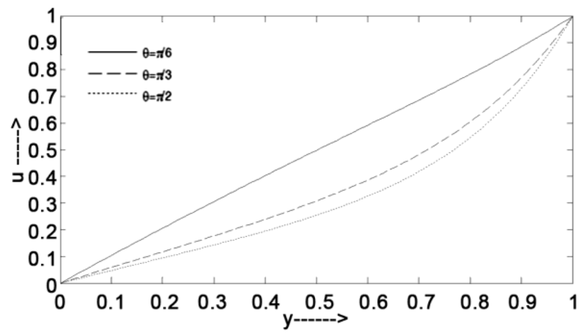

Fig. 6 — Effect of θ on velocity profile u against y

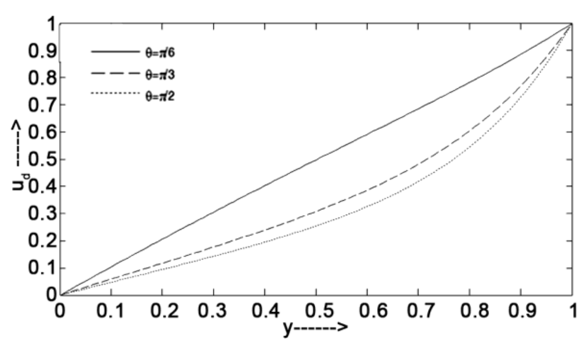

Fig. 7 — Effect of θ on velocity profile u_d against y

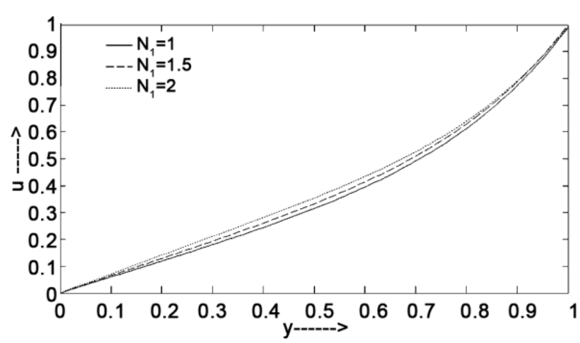

Fig. 8 — Effect of N_1 on velocity profile u against y

Figures 10 & 11, represent the velocity profile of dusty fluid and dust particles with the variation of porous medium shape factor parameter(s_1). The permeability of porous medium decreases with

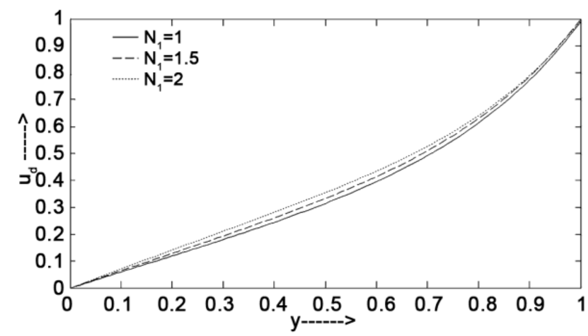

Fig. 9 — Effect of N_1 on velocity profile u_d against y

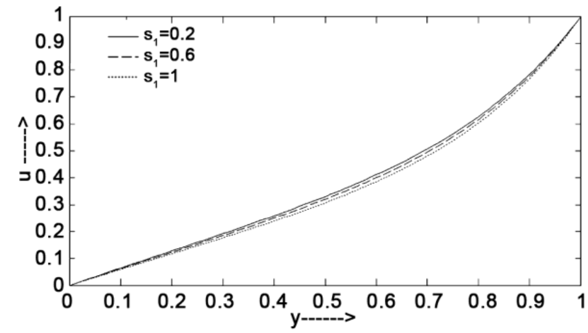

Fig. 10 — Effect of s_1 on velocity profile u against y

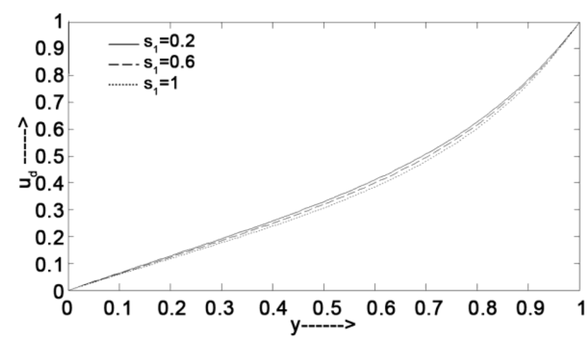

Fig. 11 — Effect of s_1 on velocity profile u_d against y

increase in s_1. Hence velocity of both dusty fluid and dust particles reduces.

Figures 12 & 13, exhibit the effect of particle concentration parameter(l_1) on velocity profile. In the flow system more and more dust particles will be distributed with rise of l_1. So, more dust particles will create disruption in the flow of fluid. Thus, velocity of both dusty fluid and dust particles decreases.

Figures 14-19, exhibit the impact of various physical parameters on shearing stress of dusty fluid (τ_1) and dust particles(τ_2).

Figures 14 & 15 represent the variation of shearing stress Grashof number(Gr) and porous medium shape factor parameter (s_1). These figures notify that shearing stress of dusty fluid and dust particles follow

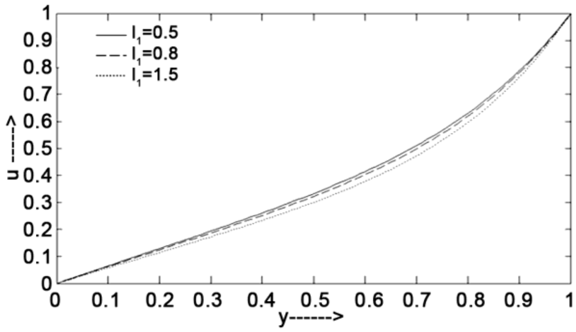

Fig. 12 — Effect of l_1 on velocity profile u against y

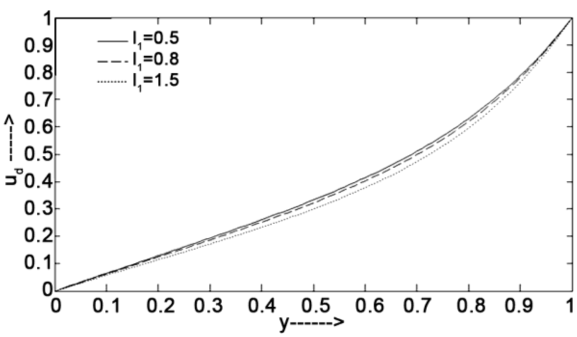

Fig. 13 — Effect of l_1 on velocity profile u_d against y

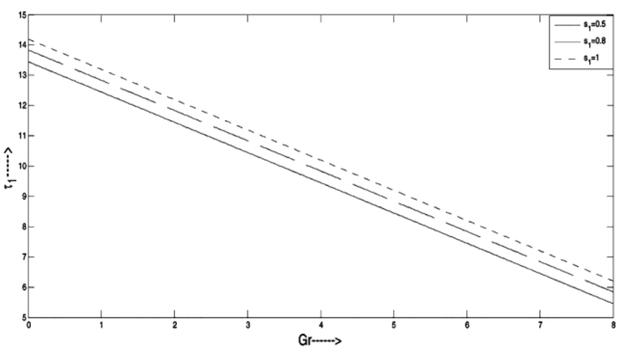

Fig. 14 — Effect of Gr and s_1 on shearing stress (τ_1)

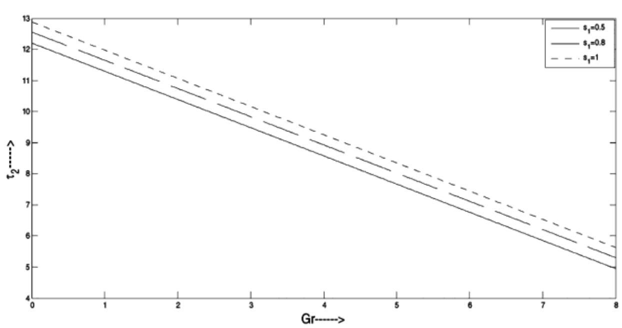

Fig. 15 — Effect of Gr and s_1 on shearing stress (τ_2)

a diminishing trend with the growth of Gr but opposite trend is noticed with the increase in s_1.

Figures 16 & 17 explain the effects of magnetic parameter(H_1) and particle concentration parameter

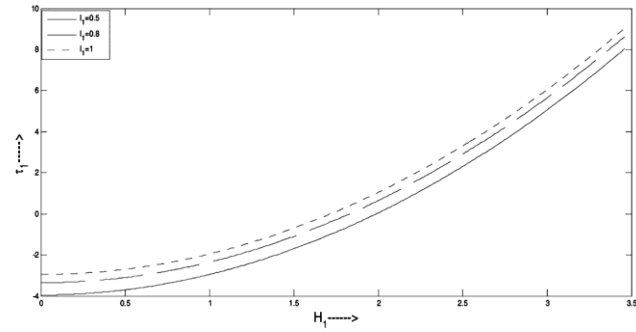

Fig. 16 — Effect of H_1 and l_1 on shearing stress (τ_1)

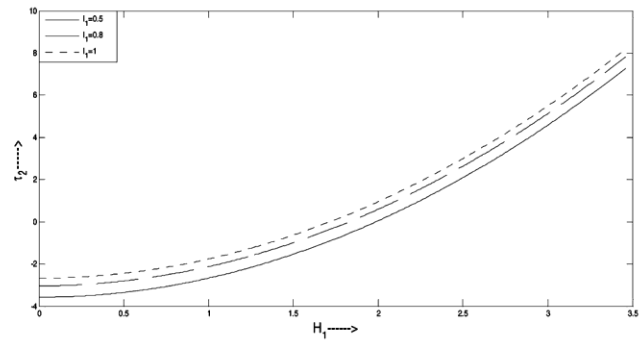

Fig. 17 — Effect of H_1 and l_1 on shearing stress (τ_2)

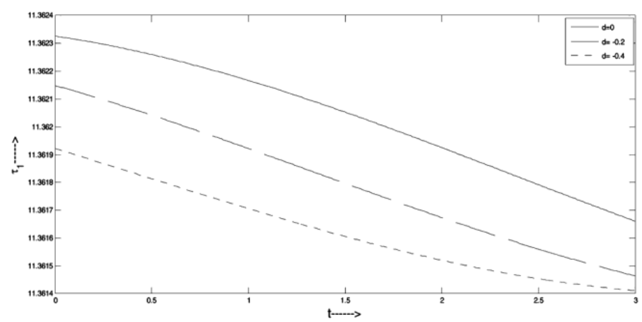

Fig. 18 — Effect of t and d on shearing stress (τ_1)

(l_1) on shearing stress. During the growth of H_1 the shearing stress of dusty fluid and dust particles follow a diminishing trend but the trend is reversed with the rise of l_1.

Figures 18 & 19 indicate the aspect of time(t) and visco-elastic parameter(d) on shearing stress of dusty fluid and dust particles. During the progress of time the shearing stress of dusty fluid and dust particles decreases steadily. But with the modification of visco-elastic parameter d=0 to d= -0.4 the shearing stress of both fluid and dust phases decreases.

Flow flux of both dusty fluid and dust particles are depicted in Figs. 20 & 21. From these figures it may be concluded that flow flux of both dusty fluid and dust particles for both Newtonian flow and non-

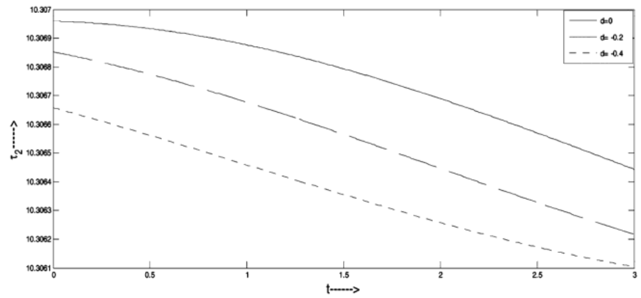

Fig. 19 — Effect of t and d on shearing stress (τ_2)

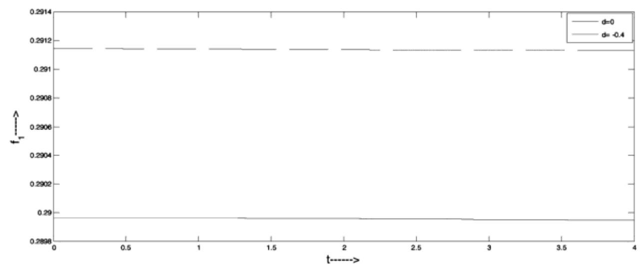

Fig. 20 — Effect of d on Flow flux of dusty fluid(f_1) against time (t)

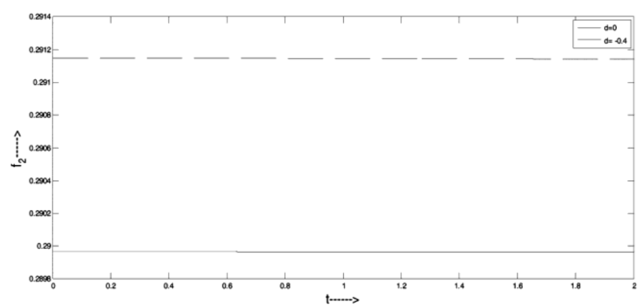

Fig. 21 — Effect of d on Flow flux of dust particles (f_2) against time (t)

Newtonian gradually decreases with time and magnitude of flow flux of non-Newtonian flow (d= -.4) is of lower order in comparison to Newtonian flow(d=0).

In this present investigation, considering visco-elastic parameter(d)=0 and angle of inclination (θ)=π/2 the obtained results are in good agreement with results obtained by Prakash & Makinde [21].

5 Conclusions

In this problem flow of dusty non-Newtonian fluid in a vertical channel in presence of inclined magnetic field has been analyzed. The governing equations of the problem are solved by perturbation technique. The obtained numerical results of velocity, shearing stress and flow flux of both dusty fluid and dust particles are presented graphically.

From this present analysis the following conclusions can be drawn

a) Increase in Grashof number and radiation parameter leads to increase in velocity of both dusty fluid and dust particles.
b) Improve in magnetic parameter, inclination parameter, porous medium shape factor parameter and particle concentration parameter reduces the velocity of both dusty fluid and dust particles.
c) The shearing stress of both dusty fluid and dust particles increases with the growth in magnetic parameter, porous medium shape factor parameter and particle concentration parameter but reverse effect has been noticed for the rise in Grashof number.
d) The shearing stress both dusty fluid and dust particles are diminishing with the progress of time and modification of visco-elastic parameter.
e) Flow flux of shearing stress both dusty fluid and dust particles decreases with time and with alteration in visco-elastic parameter.

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