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Dielectric mixing model for the estimation of complex permittivity of wet soils at C and X band microwave frequencies

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A dielectric mixing model has been developed to calculate the complex permittivity of wet soils at 5.65 GHz (C-band) and 9.5 GHz (X-band) microwave frequencies. The model considers the complex permittivity of dry soil as initial parameter. The complex permittivity of wet soil has been estimated in terms of the complex permittivity of dry soil, the complex permittivity of water at a given microwave frequency of measurement and a parameter dependent on the soil texture. The estimated values are compared with the measured values as well as with the values estimated using two well known models i.e., Wang and Schmugge model and the Hallikainen *et al*. model. The results are found to be in very good agreement with the measured values.

Keywords: Dielectric, Permittivity, Soil, Moisture, Microwave frequency

1 Introduction

Soil moisture plays an important role in affecting plant growth and yield. When soil moisture is high enough, the transpiration and photosynthesis of plants occur more efficiently and a greater mass of nutrients is available to plants through dissolution, and hence the plants can grow well[1]. The amount of air content and gas exchange in the soil is governed by the amount of water present in the soil, which affects the respiration of roots, chemical reaction in the soil, as well as the activity of microorganisms.

Complex permittivity of soil is a function of moisture content, texture structure and frequency of measurement[2] at given temperature which is very much useful in agriculture, meterology, hydrology, and in remote sensing applications. Several experiments have been conducted to estimate the complex permittivity of dry and wet soils using different methods and different types of instruments[3,4]. Further, several working models have been developed for the calculation of complex permittivity of the wet soils in terms of texture structure, frequency of measurement, moisture content, bound and free water permittivity and bulk dry density of soil at given temperature[2,3,5].

By using empirical model proposed by Wang and Schmugge[2], the complex dielectric constant ε of soil-water mixture in terms of direct mixing of the dielectric constants of constituents has been calculated. The equations presented in the model relate the complex dielectric constant of ice, water, rock and air, with the transition moisture depending on the texture structure, porosity of the dry soil and the actual volumetric moisture content in the soil. Further, the equations include the term relating the dielectric constant of initially absorbed water ε_x with the complex dielectric constant of ice and water, along with the adjustable parameter γ which can be chosen to best fit the calculated values of complex permittivity of wet soil with the experimental data. The second semi-empirical model uses the complex indices of refraction of the constituents which are mixed to give the resultant refractive index of the soil water mixture. For the moisture contents below the transition moisture, the water in the soil behaves like ice and, hence, the dielectric constant (or refractive index) for ice is used in the mixing, while above transition moisture,the dielectric properties of liquid water are used. Both models were used to calculate the complex permittivity of dry and wet soils for various moisture contents in the soils.

Hallikainen *et al*[3]. presented empirical polynomial expressions to generate ε' and ε'' as a function of volumetric moisture content (Wv) and soil texture over the frequency range 1.4-18 GHz. At each frequency, the individual polynomials were then combined into a single polynomial that expresses complex permittivity (ε', ε'') as a function of Wv, S and C; where S and C are the sand and clay textural components of a soil in per cent by weight, respectively. The constants appearing in the polynomial expressions vary depending on the frequency of measurement, and are also provided in the paper[3]. In our earlier paper [4], we compared our experimental results at C-band and X-band microwave frequencies, with the Wang and Schmugge model[2] and Hallikainen *et al*[3]. model, and found the models to be very much useful for verification of experimental values of ε' and ε''. It was observed that ε"values level off at higher moisture contents. Further, for dry soils the initial values of ε' and ε" do not match well for some of the soils. Thus, it was found to be necessary to have a simple dielectric model which uses the complex permittivity of dry soil as initial parameter and then evaluate the dielectric constant and dielectric loss of soils for various moisture contents depending on the texture structure of the soil at given frequency of measurement.

The adsorbed cations in a dry soil are tightly held by negatively charged particle surfaces composed of clay[6]. Excess cations and their anions are present as salt precipitates in the soil. In the presence of water, the salt precipitates dissolve and go into the solution. Further, the adsorbed cations partially diffuse into the solution adjacent to the particle surfaces. The electrostatic field defined by the particles inhibits this diffusion which results in a charge distribution defined by Boltzmann and Poisson equations[6]. At distance of order of 3-10Å adjacent to the hydrophilic soil-particle surfaces, a finite number of cations are closely packed, called Stern layer. Beyond the Stern layer, the concentration of cations decreases exponentially with increase in distance away from soil particle surface, called Gouy layer. Dobson *et al*[6]. proposed two dielectric mixing models for the calculation of complex permittivity of dry and wet soils depending on the texture structure and frequency of measurement. They are (i) a theoretical four-component mixing model, and (ii) a semi-empirical model.

The four component dielectric mixing model considers the soil-water medium as a host medium of dry soil solids containing randomly oriented and randomly distributed disc-shaped inclusions of bound water in the Stern layer, bulk water in the Gouy layer, and air. The model predicted $\varepsilon'_{\mathrm{calc}}$ values were approximately equivalent to $\varepsilon'_{\mathrm{meas}}$ at low moisture contents[6], but for higher moisture contents ($W_v \geq 0.3$ g cm^{-3}) and for frequency greater than 4 GHz, the $\varepsilon'_{\mathrm{calc}}$ values were less than $\varepsilon'_{\mathrm{meas}}$. For silty clay, $\varepsilon'_{\mathrm{calc}}$ agrees well with the $\varepsilon'_{\mathrm{meas}}$ at low frequencies, but the model under predicts ε'_{meas} values with increase in frequency[6] for $W_v \geq 0.3$ g cm^{-3}. Further $\varepsilon''_{\mathrm{calc}}$ was observed to be greater[6] than $\varepsilon''_{\mathrm{meas}}$ for $W_v \leq 0.2$ g cm^{-3}. Dobson *et al*[6]. also suggested a semi- empirical model in terms of soil texture, bulk density of soil (ρ_b), specific density of soil (ρ_s), volumetric moisture content and complex permittivity of free as well as bound water. It has been observed that the model [6] predicted values ε_{calc} were more linearly dependent upon the volumetric moisture content than the measured values. Further the model over estimates values at low moisture contents and under estimates ε values at higher moisture contents.

The model developed by Boyarskii *et al*[5]. calculated effective permittivity of wet soils by estimating the dielectric properties of bound water dependent on frequency of interest, as well as the textural composition of the soil. The model is based on the following facts:

(i) Water in soil remains bound when soil wetness increases from zero to a certain volume.
(ii) Change in volume of bound water in the soil leads to the change in its dielectric properties as bound water molecule relaxation time changes.
(iii) At certain wetness of soil, the dielectric properties of bound water in it become similar to dielectric properties of free water. Further increase of wetness has no impact on soil bound water dielectric constant which remains equal to free water dielectric constant.

Based on the relaxation time of bound water, the thickness 'h' of water film covering soil particles was estimated for which τ_{bw} becomes equal to τ_w of free water. According to the model, at $h \geq h_{10}$ (h_{10} ~ height of 10 water molecular layers above soil particle surface), the authors assumed $\tau_{bw} = \tau_w$. The effective permittivity $\varepsilon_{\mathrm{eff}}$ was calculated using bound and free water permittivity values, the concentration, size and permittivity of the sand, silt and clay particles in the soil.

For TDR measurements, the relationship between volumetric moisture content and effective permittivity of soil, also called a calibration curve, is needed. Generally, the empirical calibration curve obtained by Topp *et al*[7]. is used for this purpose. Miyamoto and Chikushi [1] verified some dielectric mixing models relating volumetric moisture content with permittivity of soil components. These are the Maxwell-De Loor (MD) model[8] and the semi-empirical model proposed by Birchak *et al*[9]. also called α-model. It has been observed [1] that for soil samples under consideration, Topp's model was not suitable for soils with low dry bulk density and organic soils. For them, the MD model was found to be flexible because it predicted the measured values for different types of soils with out fitting parameters.

The dielectric constant is a convenient physical parameter to describe the surface soil moisture[10]. Rayleigh developed dielectric mixing model[10, 11] for the calculation of effective dielectric constant of moist soil assuming that the soil particles are spherical in shape and there is no interaction between the particles. Behari and Sharmendra[10] modified the Rayleigh model considering that the soil particles may not be spherical, but of arbitrary shape. Further, the water fraction in the calculation of effective dielectric constant was also split in to two parts as free water (X) and bound water ($1-X$). The model contains three free parameters u-depending on the shape of the soil particles, v-depending on the filling factor which could be different for sparse and dense mixtures and X. They used the experimental data[3] at 1.4, 5 and 18 GHz to fit the unknown parameters in the modified Rayleigh mixing model. But only the calculation of real part of dielectric constant was carried out. The dielectric loss is also an important parameter in estimation of conductivity and hence salinity of the soil.

2 Experimental Details

Soil samples of different type were collected from different regions of Gujarat state, India. Table 1 presents the texture structure of the soils for which the measurements were carried out. After drying the soil samples, distilled water was added to the soil and allowed to saturate for 24 h. As the days went on, the moisture content in the soil has decreased and the corresponding measurements of permittivity were carried out using the microwave bench set up. For various moisture contents in the soils, the dielectric constant and dielectric loss were measured at 5.65 GHz (C-band) and 9.5 GHz (X-band) microwave frequencies, using the two-point method[4,12]. The experimental set- up for the two-point method is shown in Fig. 1. First with no dielectric in the short circuited sample holder, the position of first minimum D_R was measured using slotted section. Now the soil sample of certain length l_ε of given moisture content was placed in the sample holder[4,12] and corresponding position of first minimum D was measured using the slotted section. The voltage standing wave ratio r was also measured for the same soil sample. The procedure was repeated for the soil sample of another length l'_ε for the same soil moisture.

The guide wavelength and reflection coefficient were calculated as:

λ_g = 2 × (distance between successive minima with empty short circuited wave-guide sample holder)

and

$$|\Gamma| = \frac{r-1}{r+1} = \text{reflection coefficient} \qquad \ldots(1)$$

The complex number $C\angle-\Psi$ was calculated using the equation:

$$C\angle-\Psi = \frac{1}{jkl_\varepsilon} \times \frac{1-|\Gamma|\times\exp(j\phi)}{1+|\Gamma|\times\exp(j\phi)} \qquad \ldots(2)$$

where $k = \frac{2\pi}{\lambda_g} =$ the propagation constant (in the empty wave-guide)

$$\phi = 2k \times (D - D_R - l_\varepsilon) \qquad \ldots(3)$$

Table 1 — Texture structure of soil samples

Location (Region)	Soil texture (%)			Soil type	Transition moisture	Density of dry soil
	Sand	Silt	Clay		(Wt) cm^3cm^{-3}	g/cm^3
Sabarmati River bed (Ahmedabad)	93	6.2	0.8	Sand	0.1708	1.48
Gandhinagar Dist.	65	31	4	Sandy loam	0.1872	1.389
Amreli Dist.	11	78	11	Silty loam	0.2228	1.1792
Valsad Dist.	7	62	31	Silty clay loam	0.2686	1.062
Palanpur Dist.	82	16	1	Sand	0.1698	1.59

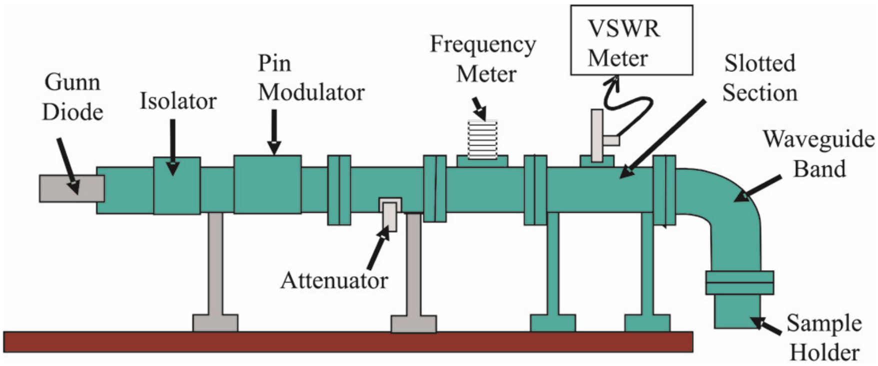

Fig. 1 — The experimental set-up for the two-point method.

Solving the complex transcendental equation:

$$C\angle -\Psi = \frac{\tanh(T\angle\tau)}{T\angle\tau} \quad \ldots(4)$$

the conductance G_E and susceptance S_E were calculated [12].

The dielectric constant ε' and the dielectric loss ε'' of the soil sample were then calculated as:

$$\varepsilon' = \frac{G_E + \left(\lambda_g/2a\right)^2}{1+\left(\lambda_g/2a\right)^2} \quad \ldots(5)$$

and

$$\varepsilon'' = \frac{-S_E}{1+\left(\lambda_g/2a\right)^2} \quad \ldots(6)$$

where a is the width of the wave-guide.

The gravimetric moisture content in the soil was calculated as[2]:

$$W_m = \frac{\text{Weight of wet soil} - \text{Weight of dry soil}}{\text{Weight of dry soil}} \quad \ldots(7)$$

Multiplying the gravimetric moisture content with the dry density of the soil (ρ_{dry}), we get the volumetric moisture content in the soil.

$$W_v = \rho_{dry} \times W_m \quad \ldots(8)$$

3 The Proposed Model

Generally, the dielectric constant of dry soils[4] varies in the range 2~4. It introduces the error in estimation of permittivity of wet soil calculated by various models which considered some fixed value of complex permittivity of dry soil. The beauty of the proposed model is that it considers the permittivity of dry soil sample as the initial parameter at the frequency of measurement. So the beginning of the permittivity curve for variation with moisture content at the frequency of measurement starts perfectly from the estimated value. Further, the model considers the mixing of permittivity of the dry soil with the permittivity of pure water at the frequency of measurement, a parameter depending on the texture structure of the soil, and the moisture content in the soil.

The permittivity of dry soil is considered as the initial parameter, which can be easily measured at any frequency using any known standard method. Hence, the error introduced in the calculation of permittivity due to (i) density variation, (ii) types of minerals present in the soil, (iii) presence of organic matter content, (iv) porosity, (v) salinity, etc., in the initial guess are avoided. Further, as the permittivity of dry soil is used in the calculation of permittivity variation for various moisture contents, the modeling becomes very simple.

The complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$ of the wet soil at C and X-band microwave frequencies is calculated using the equation:

$$\varepsilon = \varepsilon_{\text{Dry}} + \left(\frac{\varepsilon_{W,MES}}{GV}\right) \times Wv + \varepsilon_{W,MES} \times Wv \times Wv\,. \quad \ldots(9)$$

where ε_{Dry} is the complex permittivity of the dry soil at the frequency of measurement, Wv the volumetric moisture content in the soil, $\varepsilon_{W,MES}$ the complex permittivity of water at frequency of measurement using Debye model and GV is the parameter dependent on the texture structure of the soil as given by:

$$GV = 2 + [S/(3\times100)] + (7\times CL/100)\,. \qquad \dots(10)$$

where S is the sand content in per cent of dry weight of the soil in cm^3 cm^{-3}, and CL is the clay content in per cent of dry weight of the soil in cm^3 cm^{-3}.

According to Wang and Schmugge[2], for the moisture content in the soil below transition moisture, most of the water molecules are tightly bound with the soil particles, and it is difficult to polarize these water molecules. Thus, the bulk of water in wet soil has smaller value of complex permittivity as compared to that for the pure (free) water. Further according to Hallikainen *et al*[3]., the amount of water contained in the first molecular layer adjoining the soil particles is directly proportional to the total specific surface area of the soil particles per unit volume of the soil. This total specific surface area of the soil particles is dependent on the soil particle size distribution and mineralogy. The clay particles having diameter $d_{CL} \le 0.002$ mm have very large specific surface area per unit volume as compared to that of sand particles of diameter $d_S > 0.05$ mm. Thus, for the same moisture content in the soil samples, the soil having higher clay content has more bound water molecules as compared to free water molecules[3,5]. In Eq. (10), we tried to incorporate the effect of bound water on complex permittivity of wet soil depending on texture structure of the soil. As clay has large specific surface area as compared to sand particles, the magnitude of parameter GV on clay content is kept 7 times dependent on the clay. Further, sand has comparatively smaller specific surface area as compared to clay particles, so the magnitude of parameter GV is kept 1/3 times for the sand content.

4 Results and Discussion

To verify the model, we used our data set of measurements carried out in the laboratory, earlier[4]. The results are published and verified by using the two well known models[2,3]. Figure 2 shows the variation of the dielectric constant and dielectric loss of the Sabarmati river bed sand and Gandhinagar district sandy loam soil with moisture content at 5.65 GHz (C-band) microwave frequency. Further, the measured values are compared with the values calculated using the three models, i.e., Hallikainen *et al.* model, Wang and Schmugge model, and the proposed model. It can be observed from Fig. 2 that the permittivity values calculated using the proposed model agree very well with the measured values. Further, the values calculated using the proposed model are in better agreement with the measured values as compared to the empirically calculated values using the Hallikainen et al. model and the Wang and Schmugge model.

Figure 3 shows the experimental values of complex permittivity ($\varepsilon'-j\varepsilon''$) of various soil samples for various moisture contents measured at 9.5 GHz (X-band) microwave frequency. Further the experimental values are compared with the empirical values

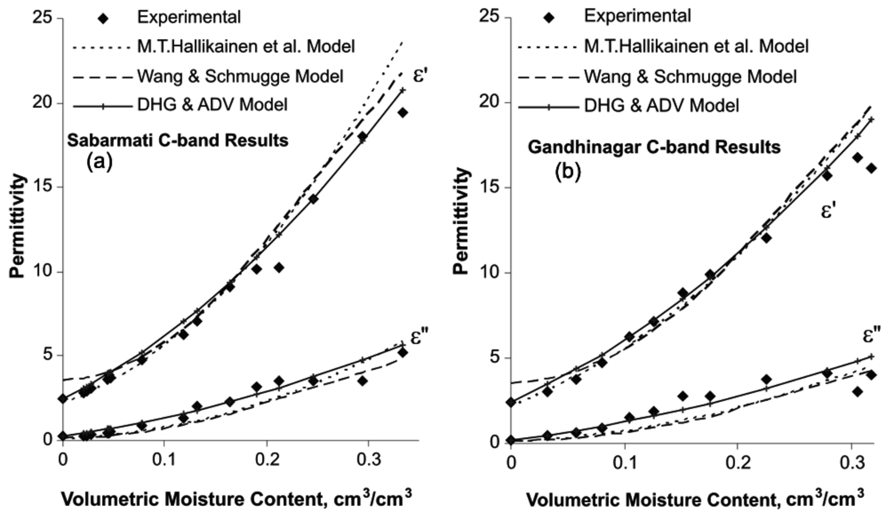

Fig. 2 — Comparison of experimental values of permittivity of the soil samples with the values calculated using the three models at C-band microwave frequency

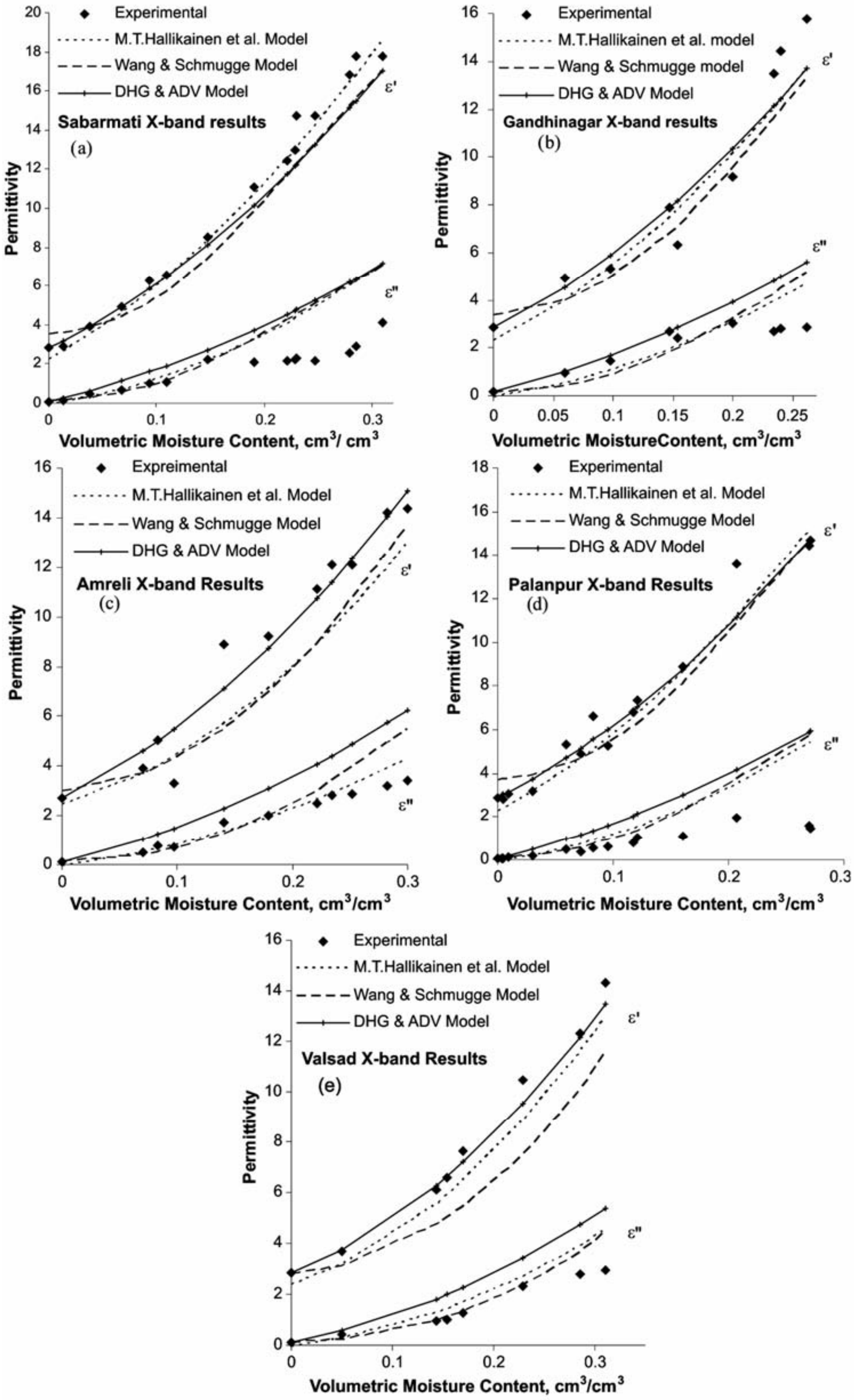

Fig. 3 — Comparison of experimental values of permittivity of the soil samples with the values calculated using the three models at X-band microwave frequency

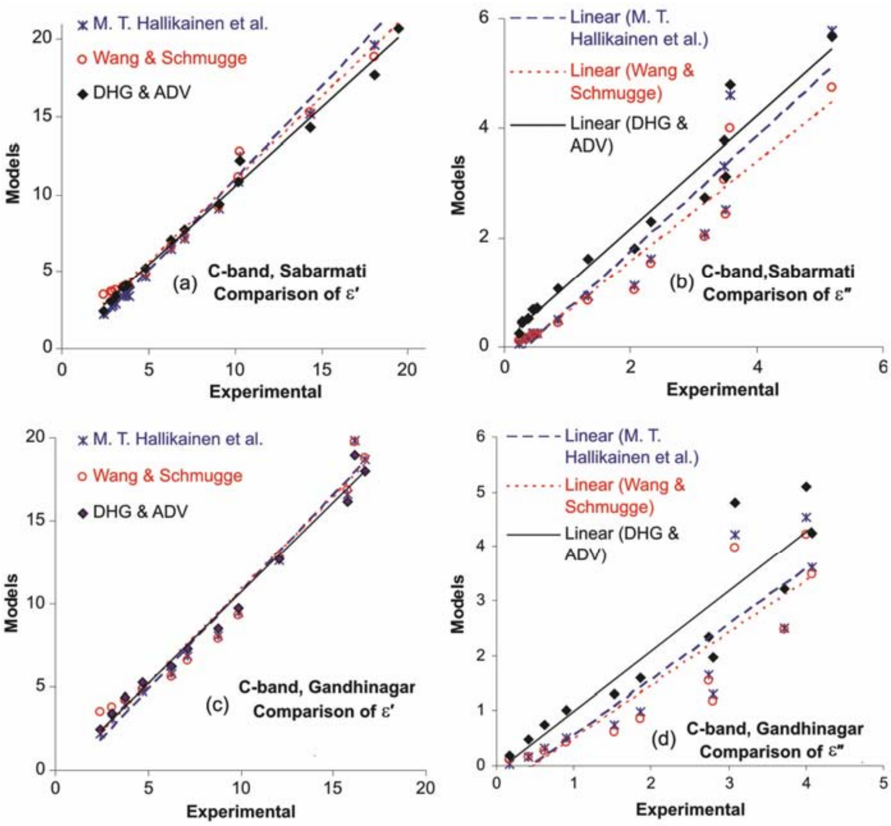

Fig. 4 — Comparison of experimental values of ε' and ε" of soils with the values calculated using the three models at C-band microwave frequency

Table 2 — Experimental values compared with the values calculated using the three models at X and C-band microwave frequencies

$$\varepsilon_{calc} = A\ \varepsilon_{meas} + B$$

Soil Sample and Type		M.T. Hallikainen *et al*. model		Wang and Schmugge model		DHG and ADV Model	
5.65 GHz (C-band)							
		ε'	ε''	ε''	ε''	ε'	ε''
Sabarmati River Sand	*A*	1.1894	1.055	1.0685	0.9168	1.0225	1.0303
	B	-0.9549	-0.375	-0.2366	-0.2779	0.2843	0.0995
	r^2	0.9904	0.9086	0.9893	0.9325	0.9917	0.9505
Gandhinagar District Sandy loam	*A*	1.1711	1.0067	1.133	0.9553	1.0922	1.0945
	B	-1.1208	-0.460	-0.602	-0.4537	-0.3163	-0.1147
	r^2	0.9795	0.7898	0.9666	0.7908	0.9853	0.8401
9.5 GHz (X-band)							
Soil Sampleand Type		M.T. Hallikainen *et al*. Model (data at 10 GHz)		Wang and Schmugge model		DHG and ADV Model	
		ε'	ε''	ε''	ε''	ε'	ε''
Sabarmati River Sand	*A*	0.9998	2.0163	0.8672	2.0374	0.8738	1.9825
	B	-0.3614	-0.469	0.5845	-0.4938	0.3727	-0.0659
	r^2	0.9909	0.9051	0.9909	0.883	0.9859	0.9136
Gandhinagar District Sandy loam	*A*	0.8328	1.4549	0.7759	1.5423	0.788	1.6571
	B	0.9343	-0.716	1.135	-0.809	1.6337	-0.426
	r^2	0.9408	0.751	0.9615	0.6825	0.937	0.7921
Palanpur Dist. Sand	*A*	1.0109	2.7238	0.8704	2.8067	0.9236	2.9707
	B	-0.5355	-0.365	0.6265	-0.398	0.3838	-0.128
	r^2	0.9675	0.822	0.9431	0.7885	0.9682	0.8553
Amreli Dist. Silt loam	*A*	0.7963	1.2183	0.828	1.5679	0.9205	1.7085
	B	0.3313	-0.278	0.2269	-0.5919	0.7135	-0.0509
	r^2	0.9421	0.9659	0.9224	0.9336	0.9492	0.9726
Valsad Dist. Silty Clay loam	*A*	0.926	1.4994	0.7694	1.4362	0.9261	1.7181
	B	-0.2933	-0.237	0.1127	-0.3535	0.3325	0.0169
	r^2	0.9928	0.9763	0.9773	0.9526	0.9934	0.9772

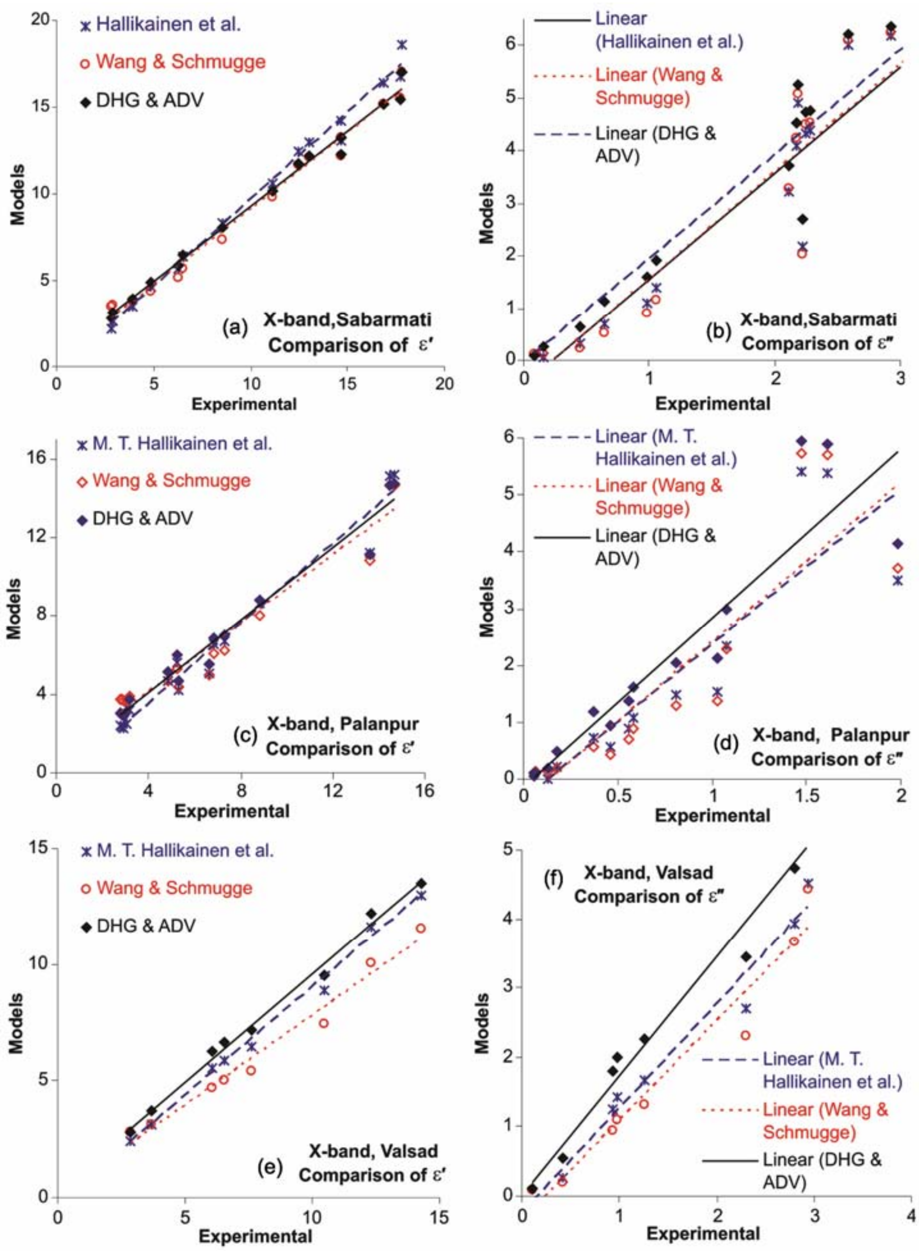

Fig. 5 — Comparison of experimental values of ε' and ε" of the soils with the values calculated using the three models at X-band microwave frequency

calculated using the Hallikainen *et al*. model, Wang and Schmugge model and the proposed model. It can be seen from Fig. 3 that the experimental values of dielectric constant ε' for various moisture contents in Sabarmati river sand are in very good agreement with the values calculated using the three models. The experimentally measured dielectric loss ε'' values of the soil for various moisture contents are in good agreement with the values calculated using the three models up to the transition moisture after which the experimental values are lower than the values calculated using the three models. This is due to the fact that the two point method is applicable to the low and medium loss dielectrics at higher moisture contents in the soil above transition moisture, the dielectric loss increases due to more free water molecules in the soil, particularly the sandy soils. Similar results were obtained for the Gandhinagar district sandy loam soil, Amreli district silt loam soil, Palanpur district sandy soil, and the Valsad district silty clay loam soil.

Comparison of experimentally measured values of complex permittivity of Sabarmati sand and Gandhinagar sandy loam soil, with the values calculated using different models, at 5.65 GHz (C-band) microwave frequency is shown in Fig. 4. The linear trend lines are drawn for the calculated values using the models as $\varepsilon_{\text{calc}} = A\varepsilon_{\text{meas}} + B$, ($\varepsilon = \varepsilon' - j\varepsilon''$), as well as with the r^2. The values of A, B and r^2, obtained from the graph for ε' and ε'' for various models are presented in Table 2. The value of the slope A near 1 for the proposed model suggests good agreement with the measured values. The intercept B should be as small as possible (near to zero) to show less deviation between the calculated and measured values of ε for linear trend. Further for ε'', the value of B is very near to zero for the proposed model, showing good agreement with the measured values. The r^2 values are also close to 1 for the proposed model, representing a very good agreement between the calculated values of permittivity with the measured values at C-band microwave frequency.

The comparison of measured values of permittivity with the values calculated using the three models at 9.5 GHz (X-band) is shown in Fig. 5. The linear trend of measured values of ε with the calculated values is also shown as $\varepsilon_{calc} = A\ \varepsilon_{meas} + B$. Corresponding values of A, B and r^2 are also presented in Table 2 for all soil types under consideration. It can be observed that the proposed model predicts the ε values in good agreement with the measured values.

5 Conclusions

The model developed by Hallikainen *et al*[3]. very well predicts the permittivity values of different types of soils for various moisture contents, but it predicts the $\varepsilon_{\text{calc}}$ values in steps of 2 GHz up to 18 GHz, since the coefficients of polynomial expressions are given at these frequencies. Thus, at intermediate frequencies $\varepsilon_{\text{meas}}$ values are supposed to be compared with $\varepsilon_{\text{calc}}$ values at nearby frequencies of the model. The Wang and Schmugge model[2] also predicts the $\varepsilon_{\text{calc}}$ values which are in good agreement with the experimental values, except $\varepsilon_{\text{calc}}$ value for dry soil is fixed.

The results obtained using the proposed model for various soil types, for various moisture contents agree very well with the experimental values at C-band and X-band microwave frequencies. The complex permittivity of dry soils of different type varies depending on their texture structure, which is an important parameter for empirical calculation of $\varepsilon_{\text{calc}}$ values of the soil for various moisture contents at different frequencies of measurement. The main flexibility of the proposed model is that it uses the actually measured complex permittivity of dry soil as the initial parameter and the texture structure dependent adjustable parameter *GV* for the given soil sample, which is used for calculation of the moisture dependent variation in dielectric constant and dielectric loss of various types of soils at given frequency of measurement.

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