Indian Journal of Pure & Applied Physics
Vol. 54, November 2016, pp. 739-743

# Empirical relation between energy and angular deviation of muons transmitted through thick slabs

Mausumi Sengupta Mitra[a]*, P K Sarkar[b], T Bandyopadhyay[a] & D N Sharma[c]

[a]HP Unit (HPD, BARC),Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Kolkata 700 064, India
[b]Manipal Centre for Natural Sciences, Manipal University, Manipal 576 104, India
[c]Health Safety and Environment Group, Bhabha Atomic Research Centre, Trombay, Mumbai, 400 085 India

*Received 5 August 2014; revised 20 July 2016; accepted 9 September 2016*

To facilitate detection of materials of high atomic number buried inside materials of low atomic number using cosmic ray muons, Monte Carlo simulations have been carried out and an empirical relation has been developed to correlate energy with the most probable angular deviation of transported muons through slabs of different materials. This empirical relation describes the correlation between the energy distribution and the distribution of angular deviation of muons passing through slabs of different materials and of different thickness, and is expected to be useful in the field of cosmic ray muon radiography.

**Keywords***:* Empirical relation, Monte Carlo simulation, Cosmic ray muon radiography

## 1 Introduction

Muons present in the cosmic ray shower are used for radiography of thick objects more effectively than X-rays or gamma-rays because of their large penetrating power. Most importantly, muon-radiography is free from unnecessary radiation hazards for the user, caused by X-rays or gamma-rays. Due to multiple Coulomb scattering, muons, while passing through any thick object, deviate in the direction and the amount of deviation depends on the atomic number *Z* and density of the material. Also, muons passing through slabs of materials suffer loss of energy, which depend on material *Z* as well as the thickness of the slab. These properties can be utilized for image reconstruction as well as material-*Z* discrimination. It is already claimed that utilizing this property of cosmic ray muons one can detect any high-*Z* material buried in low-*Z* surroundings, which is again useful in surveillance of cross border transport of nuclear and other heavy materials[1]. Therefore the high-energy cosmic ray muons (mean energy at sea level 3-4 GeV) may provide a way of discriminating materials of different densities and may be used to detect high-*Z* material in the presence of materials of low-Z. Recently there are several claims of utilizing muon imaging technique for imaging nuclear fuels inside reactor[2] which may be beneficial for investigation of the damaged core of reactors like those in Fukushima[3].

Muons passing through a material lose energy through different processes. The main processes of muon energy loss are: atom excitation and ionization (i.e., collision losses), bremsstrahlung, electron-positron pair production and inelastic scattering (photo-nuclear interaction). Collision losses are described by cross-sections comparable to atomic dimensions and occur very frequently along the path of any ion. Since energy loss per collision is small (about 10 eV), the resulting angular deviation per collision is also very small. This process of multiple scattering does not contribute much to the muon energy loss, so it is usually neglected in considering muon energy degradation. The other three interactions occur when the muon passes close to the nuclear protons and hence these are relatively rare, but since they involve much larger energy losses (about a few MeV) the muon scattering angle is quite large even in a single event. The mean free path for these large momentum transfers is called the radiation length, which decreases with increasing atomic number and density. These processes produce muon deflection from the initial direction and the resultant scattered angle depends on the radiation length of the material. Therefore, the *Z*-dependent distribution of angular

*Corresponding author (E-mail: mausumi@vecc.gov.in)

deviation plays important role in muon radiography[4]. Also there are claims that energy loss estimation of cosmic ray muon while transported through any object may provide some information useful in muon radiography. Jenneson *et al*[5] have carried out some measurements and have proposed a technique of imaging the internal structure of large vessels by using cosmic-ray muon energy loss estimations. These authors have indicated that the energy loss measurement technique is a promising one and can be extended for material identification, though the proposed method faces several practical difficulties of muon energy measurements before and after propagation through the material, and this is further complicated by the cosmic ray muon energy distribution.

In an earlier reported work[4], the possibility of detecting the presence of lead and uranium buried inside large quantities of iron has been demonstrated using an empirical relation of angular deviation of muons transmitted through slabs of different materials. It was observed, however, that difference between lead and uranium was not discernible in all the cases when measurement errors were considered. It is anticipated that additional information on the correlation, between angular deviation and energy degradation of transmitted muons, might provide a way of more accurate discrimination of materials having closer atomic numbers.

In this paper Monte Carlo simulations have been carried out to estimate the angular deviation (from the incident direction) as well as the energy loss distribution of muons passing through different materials and an attempt has been made to correlate the maxima in the distribution of angular deviation with the maxima of energy loss distribution to facilitate material discrimination.

## 2 Simulation Details

To simulate the cosmic ray muon shower, parallel monoenergetic beam of muons of different energies in the range 1 GeV to 5 GeV (most significant part of the cosmic ray muon spectrum) in steps of 1 GeV, have been considered to be incident normally on the slabs of different thickness and of different materials. The aim is to construct a Green's function using the set of results obtained with monoenergetic beams, so that for any measured spectrum of cosmic-ray muons one can obtain the desired energy distribution. Slabs of uranium, lead and iron of different thickness ranging from 0.1 m to 0.5 m in steps of 0.1 m were considered. The spectrum of cosmic ray muons at sea level has its peak around 3 GeV. The Muon Simulation Code (MUSIC)[6-9] has been used, for this study, in particular the version developed for muon propagation through small thicknesses of materials[9].

### 2.1 The code MUSIC

The Monte Carlo based code MUSIC was developed initially to simulate propagation of muons through rocks of thickness about a kilometer or more for underground cosmic ray experiments. The standard version of the code MUSIC deals with muons transport through large thickness of matter. Another version of the code MUSIC is known as the 'thin slab version', which can be utilized to simulate propagation of muons through small thickness of materials. We have used the second version of the code MUSIC. In the MUSIC code energy losses due to ionization, bremstrahlung pair production and muon-nucleus inelastic scattering are taken into account. The path of muons between two stochastic interactions is sampled according to the total muons interaction cross section. Then the process of interaction is chosen according to the relative contribution of various processes to the total cross section. Finally, the energy loss of muons in the interaction is selected according to the differential cross section of the chosen process. Such energy loss is considered continuous if it is below a cut off energy and stochastic if it is more than the cut off energy. For thin slab version of the MUSIC code the cut off energy is taken as 1 MeV. The ionization energy loss is calculated for this special version using Landau distribution. The deflection of muons due to multiple coulomb-scattering in a plane perpendicular to the initial direction is treated according to the Gaussian approximation. The projection of muon angular deviation and lateral displacement on two orthogonal planes that include the initial direction of muons are also considered. The muon angular deviation due to multiple scattering is shown schematically in Fig. 1. The angular deviation due to inelastic scattering is computed using double differential crosssections[6].

## 3 Results and Empirical Fitting

Figure 2 shows a plot of the energy distribution of muons transported through slabs of Fe, Pb and U, each of 0.5 m and 1.0 m thick for incident muon energy of 3 GeV, while Fig. 3 shows the plot of the distribution of angular deviation for the same

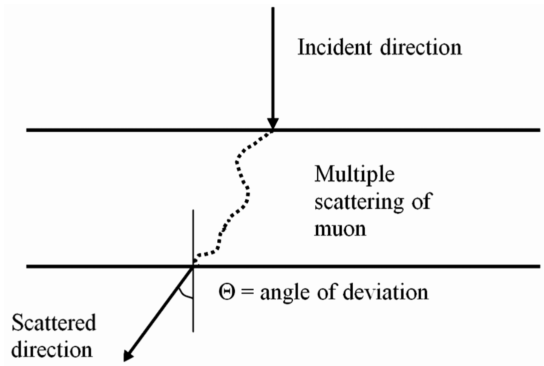


Fig. 1—Scattering of muons through material and angle of deviation at the exit surface. The magnitude of scattering is exaggerated for this illustration

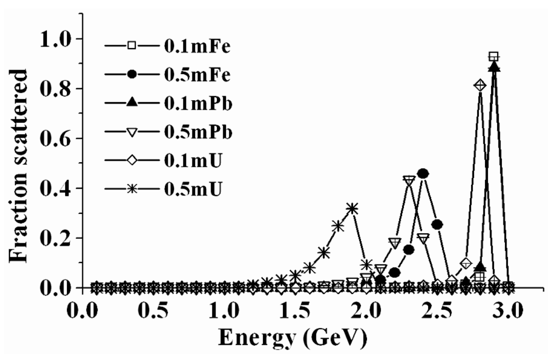


Fig. 2—Energy distribution of 3GeV muons transmitted through slabs of different materials calculated using the MUSIC code

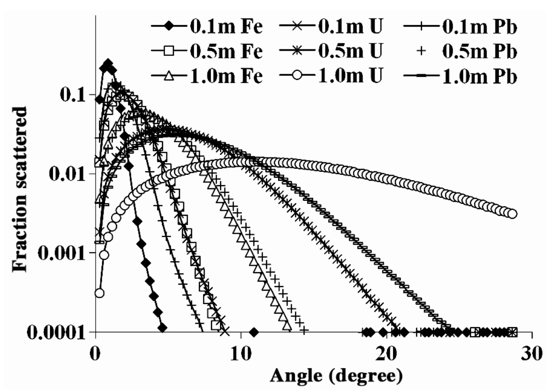


Fig. 3—Angular distribution of 3 GeV muons transmitted through slabs of different materials

materials. For each of the three materials (Fe, Pb and U) and for 0.3 m slab thickness the peak position of the angular distribution $A_{peak}$ is plotted in Fig. 4 against the peak position of the energy distribution $E_{peak}$ for the transmitted monoenergetic muons in the range 1-5 GeV.

The symbols are the values obtained from Monte Carlo simulations with error bars indicating the uncertainties in both energy and angular

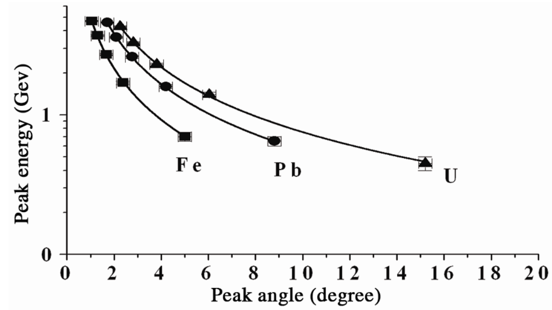


Fig. 4—Peak positions on angular deviations plotted against peak positions on energy distributions with error bars for different monoenergetic muons transmitted through 0.3 m of iron, lead and uranium. The solid lines represent the result of our empirical fitting while the symbols with error bar represent the values calculated using MUSIC code for iron, lead and uranium

peak positions. These errors are statistical errors arising out of Monte Carlo simulations. The uncertainties indicated do not contain any systematic error that might have been contributed by the inadequacy of model employed or inaccurate cross section data used in the simulation. The solid lines are the fitted curves giving empirical relations between angular peak $A_{peak}$ and energy peak $E_{peak}$ positions that are parameterized as:

$$A_{peak} = a(E_{peak})^{b} \quad \ldots (1)$$

where, $b$ = -1.2$\pm$ 0.01 for all materials and for all thickness. The chi-square values of the fits range between $10^{-2}$ and $10^{-3}$.

The parameter $a$ can be fitted as a quadratic function of the thickness of the material as:

$$a = p + qx + rx^2 \quad \ldots (2)$$

where, $x$ denotes thickness of the material. Figure 5 gives a plot of the parameter $a$ against $x$ (thickness) for iron (Fe), lead (Pb) and uranium (U) with error bars.

The values of coefficient of determination $R^2$ obtained in these cases are about 0.99. The curves are non linear and runs almost parallel to each other. It can be inferred from Figs 4 and 5 that for a fixed peak position in the energy distribution, the peak position in the angular deviation will be largest for uranium followed by lead and iron.

We have found that the parameter $p$ in Eq. (2) is a linear function of the material density ρ (g $cm^{-3}$):

$$p = m_1\rho + c_1 \cdot \quad \ldots (3)$$

where, we have found $m_1$=0.148 and $c_1$=1.115. Figure 6 gives a plot of the parameter $p$ against the density ρ of the material. In this case the value of $R^2$ is about 0.99.

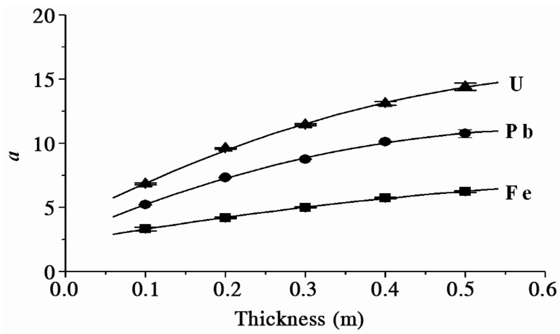


Fig. 5—The values of parameter *a* are plotted against thickness of iron, lead and uranium. The solid lines represent the result of our empirical fitting while the symbols with error bar represent the values of parameter *a* obtained from the previous fitting

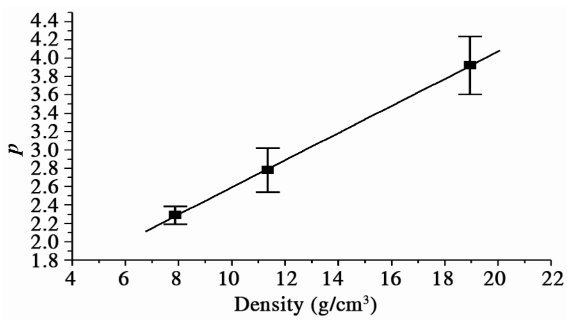


Fig. 6—The values of parameter *p* are plotted against density of different materials. The solid lines represent the result of our empirical fitting while the symbols with error bar represent the values of parameter *p* obtained from the previous fitting

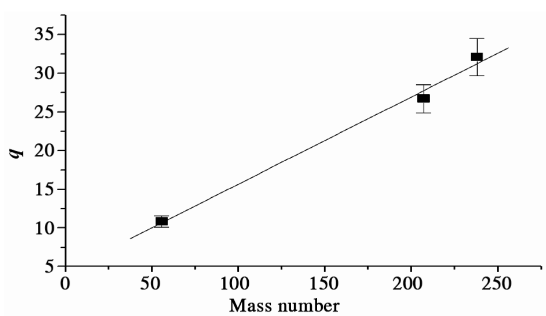


Fig. 7—The values of parameter *q* are plotted against mass number of different materials. The solid lines represent the result of our empirical fitting while the symbols with error bar represent the values of parameter *q* obtained from the previous fitting

The other parameter $q$ in Eq. (2) is found to be a linear function of the mass number $A$ of the material:

$$q = m_2 \rho A + c_2 \quad \ldots (4)$$

with $m_2$ = 0.113 and $c_2$ = 4.356. A plot of $q$ against mass number is given in Fig. 7. The value of $R^2$ is about 0.99.

The third parameter $r$ is a linear function of the atomic number $Z$ as:

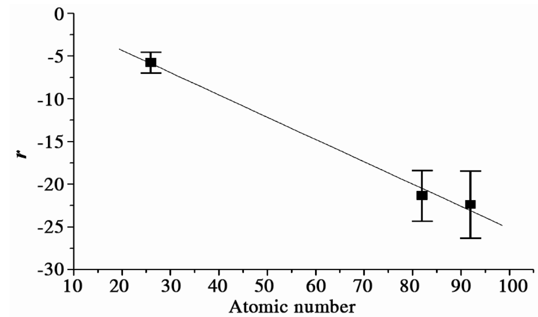


Fig. 8—The values of parameter *r* are plotted against atomic number of different materials. The solid lines represent the result of our empirical fitting while the symbols with error bar represent the values of parameter *r* obtained from previous fitting

$$r = m_3 Z + c_3 \quad \ldots (5)$$

with $m_3$ =-0.2609 and $c_3$= 0.920. The value of $R^2$ in this case is 0.99 and a plot of $r$ against atomic number $Z$ is given in Fig. 8.

In an earlier work[4] the authors have established an empirical formulation to calculate the angular deviation of muons transmitted through thick slabs of different materials. Using this formulation they have shown that two ratios, namely, *r*1 and *r*2 of the number of transmitted muons deviated at different directions from the original direction can be effectively used as a good indicator for discriminating muons transmited through high-*Z* and low-*Z* materials. Here, *r*1 is the number of muons deviated between angles 0.5º and 2.5º divided by the number of muons deviated between 2.5º and 25º while *r*2 is the number of muons deviated between 0.5º and 10º divided by the number of muons deviated between 15º and 25º It has been demonstrated that using this technique discriminations between U and Fe as well as between Pb and Fe for slabs of any thickness can be effectively done when the measurement errors are within about 15%. However, this technique failed to discriminate between 25 cm thick slab of U from 38 cm thick slab of Pb when the measurement error is more than 5%.

In this work a correlation between the maximum (peak) of the distribution of angular deviation and the maximum of the energy distribution of transmitted muons is obtained and an empirical relation between the two has been established. Now with the possibility of energy measurement in addition to the measurement of angular deviation this empirical relation can be used to discriminate between different materials more accurately compared to the technique based only on measurement of angular deviation.

Table 1—The parameter *a* for Pb and U at different slab thickness

| Thickness (cm) | *a* for Pb | *a* for U |
|---|---|---|
| 25 | 8.121 | 10.543 |
| 38 | 9.844 | 12.882 |
| 46 | 10.545 | 13.946 |

This can be demonstrated using the case of discrimination between 25 cm of U and 38 cm of Pb. This could not be resolved using the earlier technique of using only the measured angular deviation data. If energies of the transmitted muons are also measured along with angular distributions and Eq. (1) is used then knowing the value of the parameter *b* (independent of the material) the value of the parameter *a* can be obtained easily. Table 1 shows values of parameter *a* for different thickness of Pb and U.

As can be seen the values of the parameter *a* for 25 cm of U and 38 cm of Pb are quite different and can be discriminated. In fact, the value of *a* for 25 cm of uranium coincides with 46 cm of lead (almost double the thickness of uranium).

## 4 Conclusions

In continuation of the existing empirical relation for angular deviation of cosmic ray muons transported through slabs of iron, lead and uranium with the incident muons energy and the thickness, atomic mass, radiation length and density of the material[4], a new empirical relation has been established that connects the most probable energy with the most probable angular deviation of the transported muons (Eq. (1)). The present empirical relation is obtained by fitting the simulation results using a Monte Carlo muon transport code specially developed for transmission through thin slabs. The results obtained with earlier reported empirical relation[4] indicate that uranium and lead can be distinguished from iron but discrimination between lead and uranium is difficult.

The proposed empirical formula and the corresponding formulae describing variations of different parameters with thickness, density, atomic number and mass number of the materials are expected to provide useful information for discriminating materials with different *Z* values. In particular, discrimination between Pb and U slabs has been demonstrated to be possible. Also, the formula is very simple with less number of parameters. However, experimental errors are expected to hamper the theoretically indicated feasibility, though the underlying relations given by the present equations are expected to remain valid unless some systematic errors creep in the measurement. Measurements are required to be done accurately and with less statistical uncertainty. Nevertheless, the present empirical formulation will prove to be helpful in the case of actual measurements.

The present theoretical analysis indicates the requirement of the angular deviation measurements to be performed with high granularity or angular resolution of the order of about 0.5 degree. This might turn out to be difficult but not impossible. It may be mentioned in this context that high angular resolution in the measurement of cosmic ray muons has been achieved in a project called GRAND[10]. The system consists of a proportional wire chamber array with four pairs of orthogonal proportional wire chamber planes located above each other with a separation of 20 cm. Each plane contains 80 cells and above the bottom pair of planes is a 5 cm thick steel plate achieving an average resolution of 0.26 degree. A suitable modification of such a system can be utilized for muon radiography utilizing the proposed technique for material discrimination.

## Acknowledgement

The authors are thankful to V A Kudryavtsev for supplying the code MUSIC.

## Reference

1 Schultz L, Borozdin J K, Gomez J J, Hogan G E, McGill J A, Morris C, Priedhorsky W, Sounders A & Teasdale M E, *Nucl Instr Meth Phys Res*, 519 (2004) 678.
2 John P, Mara A & Jeffrey B, *J Appl Phys*, 113 (2013) 184909.
3 Konstantin B S G, Lukic Z, Milner E, Miyadera H, Morris C & Perry J, *Phys Rev Lett*, 109 (2012) 152501.
4 Mausumi S M, Sarkar P K & Kudryavtsev V A, *Nucl Instr Meth Phys Res*, 604 (2009) 684.
5 Jenneson P M, Gilboy W B, Simons S J R, Stanley S J, Rhodes D, *Chem Eng J*, 130 (2006) 75.
6 Antonioli P, Ghetti C, Korolkova E V, Kudryavtsev V A & Sartorelli G, *Astropart Phys*, 7 (1997) 357.
7 Kudryavtsev V A, Korolkova E V & Spooner N J C, *Phys Lett B*, 471 (1999) 251.
8 Sokalski I A, Bugaev E V & Klimushin S I, *Accuracy of Muon Transport Simulation*, Proceedings of 2$^{nd}$ Workshop on Methodical Aspects of Underwater/Underice Neutrino Telescopes, Hamburg, 2001.
9 Kudryavtsev V A, Comput Phys Commun, 180 (2009) 339.
10 Poirier J & Andrea C D, *A Measurement of Secondary Muon Angular Distribution*, Proceedings 27$^{th}$ International Cosmic Ray Conference (ICRC), Hamburg, 07-15 August (2001); arXiv:astro-ph/0109490v1, 2001.