



Simulative Comprehensive Analysis of 2D Algebraically Constructed Codes for Optical CDMA

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In this manuscript, comprehensive analysis of distinct optical codes belonging to family of algebraically constructed codes has been demonstrated. Many optical codes belonging to algebraically constructed family such as Extended Reed-solomon codes (E-RS), Multi-level prime codes (MPC), Extended-multi level prime codes (E-MPC), and Variable-weight quadratic congruence codes (VWQCC), Bipolar/Unipolar codes are taken into consideration. The performance characteristics of these codes are examined through Hard-error limiting probability (HEP) equations. Investigations revealed that Bipolar/Unipolar code is the one algebraically developed code that has outperformed all the codes considered here even in the presence of Multi-user interference effect. It has been observed that this optical 2D Bipolar/Unipolar code shows minimum BER value with increasing user count. Hence Bipolar/Unipolar code is robust to multiple user interference affected OCDMA systems.

Keywords: Optical-CDMA, E-RS, MPC, E-MPC, VWQCC

1 Introduction

Optical code division multiple access (OCDMA) refers to making use of coding in optical domain to add the features of CDMA like security and optical domain like huge bandwidth. Owing to its attractive properties, enormous research is going on this from past thirty-forty years¹⁻⁴. Mainly two types of OCDMA are, “synchronous” and “asynchronous”. Synchronous OCDMA is based on synchronization and asynchronous utilize the orthogonal properties of optical codes that makes it more practical in case if multiple subscribers using the system at a time. Simultaneous users accessing the system at same time lead to multiple access interference (MAI). With increase in number of users, MAI also increases.

Due to this MAI, optical codes are constructed that aims to provide good quality of system even in presence of MAI with large number of users. Due to some drawbacks of 1D code; new 2D optical codes were evolved after loads of research. 2D code supports large number of users with high efficiency. In addition to this, 2D codes inherently own a feature of flexible code construction with higher cardinality. By employing two domains: frequency and time/wavelength and time in procedure for construction of code; system capacity for accommodating more number of subscribers increased immensely¹⁻³.

Construction method for various code families differ from family to family. But among those families there exists one code construction procedure which is simpler and flexible as compared to others. By employing finite fields code construction became way simpler and more presentable. Algebraic code construction can be applied to easily apply to prime sequences, OOCs and Congruence codes *etc.* General algebraic construction is usually based upon a generating finite algebraic field over a specific prime number. In this manuscript, various 2D codes that belong to algebraically constructed family are analyzed to suggest best performing code. Optical codes that are designed over finite fields like Galois field constructions this paper comprises of four sections. Section 1 consists of brief introduction to OCDMA. Section 2, describes the design and features of codes under analysis. Section 3 provides the results and discussions after analyzing the family of codes. Finally section 4 concludes the manuscript.

2 Design and Features of Codes

Algebraic codes provide a flexible yet simple approach to construct the codes. By employing finite fields like Galois field over a prime number, code generation becomes simple because then easier arithmetic and mathematical operations can be applied. The basic construction procedure in this family employs polynomial and matrix representation

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of information and encoding schemes introduces simplicity in codeword evaluation and representation.

Some of the algebraically constructed code shows great flexibility in choosing between codeword count and performance in accordance to applications. Hence providing choice in response to requirement. Also a multi-level architecture or structure of codeword can be generated whenever one wants to choose number of codeword from whichever set he wants. Usage of Galois field in code construction of some codes led to easy indexing of codeword by manipulating the parameters involved in the construction method.

A short description of construction procedure along with properties of each code is given as follows.

E- RS

Construction of 2-D optical E-RS based code starts with Galois field over prime number selected. Then after evaluating Generator polynomials, Minimal polynomial and then simple multiplication with message signal coefficients to obtain codeword coefficients ends up the construction⁵. These code provide flexibility of selecting cross correlation value to obtain either larger cardinality by relaxing λ_c or by keeping it close to zero for obtaining less MAI to have better code performance.

HEP: The HEP equation is given below 5:

$$Pe = \frac{1}{2} \sum_{i=0}^w (-1)^i \binom{w}{i} \left[\sum_{j=0}^{\lambda_c} q_{\lambda_c, j} \frac{\binom{w-i}{j}}{\binom{w}{j}} \right]^{K-1} \dots (1)$$

Here $q_{\lambda_c, j}$ is

$$q_{\lambda_c, j} = \frac{1}{2} \times \frac{h_{\lambda_c, j}}{p(p^{\lambda_c} - 1)} \dots (2)$$

Here K stands for Number of simultaneous users, $\frac{1}{2}$ depicts for equal probability of ‘1’ and ‘0’ in OOK. In denominator p denotes number of possible shifts in a codeword whereas $p^{\lambda_c} - 1$ represents possible number of p^{λ_c} codeword.

Multi-level Prime codes

These 2-D optical codes are constructed using Finite field GF (p)⁶. For a selected prime number p and a positive integer n and w, each set of code words represented by code matrices is denoted by:

$$S_{i_n, i_{n-1}, \dots, i_1} = (S_{i_n, i_{n-1}, \dots, i_1, 0}, S_{i_n, i_{n-1}, \dots, i_1, 1}, \dots, S_{i_n, i_{n-1}, \dots, i_1, j}, \dots, S_{i_n, i_{n-1}, \dots, i_1, w-1}) \dots (3)$$

Where jth element is given by:

$$S_{i_n, i_{n-1}, \dots, i_1, j} = (i_n j^n) \oplus_p (i_{n-1} j^{n-1}) \oplus_p \dots \oplus_p (i_1 j) \dots (4)$$

Construction of Multi-level prime codes based upon these prime sequence and these code words can be arranged in sub-set which further is arranged into multiple-levels and hence called as Multi-level prime codes.

HEP: The HEP equation is given as follows:

$$Pe = \frac{1}{2} \sum_{k=0}^w (-1)^k \binom{w}{k} \left[\sum_{j=0}^{\lambda_c} q_{\lambda_c, j} \frac{\binom{w-k}{j}}{\binom{w}{j}} \right]^{K-1} \dots (5)$$

$$q_{n, j} = \frac{1}{2} \times \frac{h_{n, j}}{p(p^n - 1)} \dots (6)$$

Here K denotes simultaneous users & K-1 refers to interfering users, $\frac{1}{2}$ depicts for equal probability of ‘1’ and ‘0’ in OOK.

EMPC

These 2-D optical codes belong to WH/TS incoherent unipolar codes. The code construction is performed from finite field i.e. Galois field GF (p). These 2-D optical EMPC increases the code cardinality by releasing cross-correlation constraints and obtaining asymptotically optimal cardinality⁷. Similar to characteristics of E-RS and MPC these code words have a Tree structure too. Likewise if cross-correlation value is increased the codeword count increases by sacrificing code performance slightly. Each code word of EMPC is represented as:

$$S_{i_{k, n}, i_{k, n-1}, \dots, i_{k, 1}, i_{k-1, n}, i_{k-1, n-1}, \dots, i_{k-1, 1}} = (S_{i_{k, n}, i_{k, n-1}, \dots, i_{k, 1}, i_{k-1, n}, i_{k-1, n-1}, \dots, i_{k-1, 1}, 0}, S_{i_{k, n}, i_{k, n-1}, \dots, i_{k, 1}, i_{k-1, n}, i_{k-1, n-1}, \dots, i_{k-1, 1}, 1}, \dots, S_{i_{k, n}, i_{k, n-1}, \dots, i_{k, 1}, i_{k-1, n}, i_{k-1, n-1}, \dots, i_{k-1, 1}, w-1}) \dots (7)$$

HEP: Toanalyse performance of these optical codes ON-OFF keying (OOK) method is used and

other effects like Shot noise, Thermal noise, Beat noise all were neglected in order to consider only effect of MAI. The HEP equation is given below 7:

$$Pe = \frac{1}{2} \sum_{i=0}^w (-1)^i \binom{w}{i} \left[\sum_{j=0}^n q_j \frac{\binom{w-i}{j}}{\binom{w}{j}} \right]^{K-1} \quad \dots (8)$$

$$q_j = \frac{1}{2} \times \frac{h_{n,j}}{p^k (p^{nk} - 1)} \quad \dots (9)$$

Where K stands for Number of simultaneous users $\frac{1}{2}$ depicts for equal probability of ‘1’ and ‘0’ in OOK. In denominator p denotes number of shifts in a codeword

VWQCC

This 2-D optical code construction based upon Quadratic congruence function, a complete set of Quadratic congruence sequence can be obtained given as follows⁸:

$$S_{i,j,k} = (s_{i,j,k}(0), s_{i,j,k}(1), \dots, s_{i,j,k}(m), \dots, s_{i,j,k}(p-1)) \quad \dots (10)$$

When this sequence is mapped into a binary code sequence then codeword set

$$C_{i,j,k} = (c_{i,j,k}(0), c_{i,j,k}(1), \dots, c_{i,j,k}(m), \dots, c_{i,j,k}(p-1)) \quad \dots (11)$$

Whereas $p^{\lambda_c} - 1$ represents possible number of p^{λ_c} code words. *i.e.* unique tree structure, symmetry in codeword set with proper auto- & cross-correlation function. Code weight can be varied which provide the flexibility to choose between codeword number and performance

HEP: The equation to determine HEP is as follows:

$$Pe = \frac{1}{2} \sum_{i=0}^{w_v} (-1)^i \binom{w_v}{i} \left(1 - \frac{iq}{w_v} \right)^{K-1} \quad \dots (12)$$

where average probability equals

$$q_v = \frac{1}{2} \times \frac{w_v(w_v - 1)}{p^2 - 1} \quad \dots (13)$$

K is number of simultaneous users. 0.5 depicts equal probability of occurrence of data bits ‘1’ & ‘0’. Numerator denotes any two code sequence will cause $w_v(w_v - 1)$ times hits & denominator denotes $p^2 - 1$ interfering code words.

Bipolar/Unipolar codes

This code is generated by using Unipolar version of Bipolar code. This W/T code is generated by

permutation of multi-wavelength code words controlled by prime sequences. To keep cross correlation constraint at most one, non-prime sequences which have repeated elements are neglected⁹.

HEP: The HEP equation is given as below:

$$Pe = \frac{1}{2} \sum_{i=0}^w (-1)^i \binom{w}{i} \left(1 - \frac{iq}{w} \right)^{K-1} \quad \dots (14)$$

where q is average hit probability which is contributed by q_0 & q_i which depicts number of 1-hits from interfering code words.

$$q_0 = \frac{w^2(\phi_{unipolar} p - 1)}{2n(\phi_{unipolar} p^2 - 1)} \quad \dots (15)$$

$$q_i = \frac{w^2(\phi_{unipolar} p - 1) + (w - 1)^2}{2n(\phi_{unipolar} p^2 - 1)} \quad \dots (16)$$

3 Results & Discussion

E-RS vs MPC and E-MPC

In Fig. 1(a) E-RS clearly outperforms MPC. For same number of prime number $p = \{31, 37, 41\}$ and

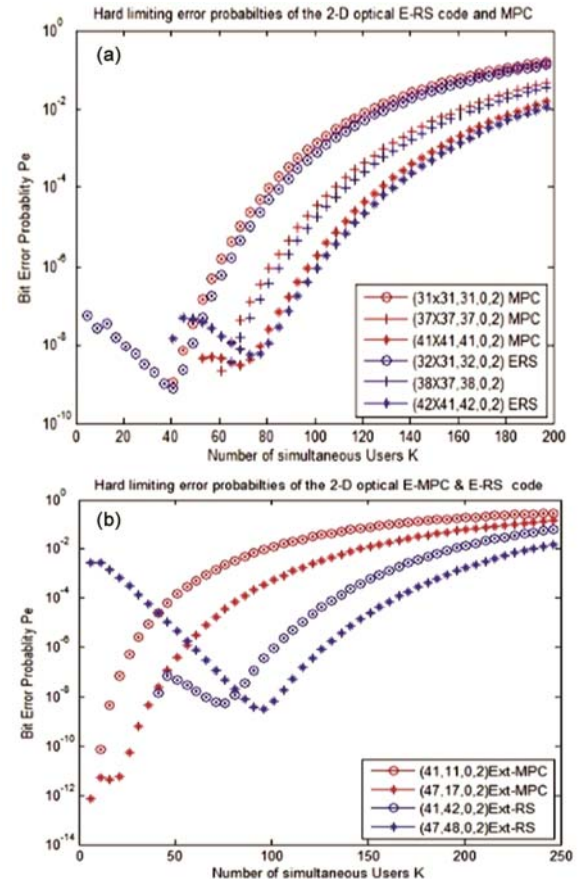


Fig. 1 — (a) E-RS vs. MPC (b) E-RS vs. E-MPC

for same code length $N (=p)$, E-RS have better properties as compared to MPC and hence initially E-RS curves are lower than MPC. However with the increase of number of users, effect of MAI starts countering this advantageous feature of E-RS and eventually MAI becomes so prominent that it almost reaches and gets superimposed with the MPC curves. In Fig. 1(b), E-RS is better than E-MPC. E-RS have better properties as compared to E-MPC and E-RS curves are always lower than E-MPC. E-RS has number of wavelengths $L (=p+1)$, heavier code weight $w (=p+1)$ as compared to E-MPC which have number of wavelengths usually variable but here $L =p$ and lower code weight $w =p$ which causes better code performance of E-RS than E-MPC.

E-RS vs VWQCC and Bipolar/Unipolar

Figure 2(a) clearly shows VWQCC is superior in performance to E-RS. Although E-RS have higher number of wavelengths $L = p+1$ than that of VWQCC

($= p$) but VWQCC have some superior properties which is the cause of its better performance than that of E-RS. VWQCC have larger code length $N=2p^2$ ($N= p$ for E-RS) along with higher value of Auto-correlation $\lambda_a = p$ ($\lambda_a = 0$ for E-RS) which eventually cause less number of hits and hence low BER. Figure 2(b) shows that Bipolar/Unipolar code completely outperforms E-RS code. For almost same LN product value or Bandwidth expansion Bipolar/Unipolar performs far better than E-RS code because it has other improved properties than E-RS.

MPC vs E-MPC and VWQCC

Figure 3(a) shows E-MPC has better code performance than MPC for same number of prime number chosen $p = \{7,11,13\}$, for same LN product ($= p \times p$), for equal code weight $w (= p)$ and similar correlation properties *i.e.* $\lambda_a = 0$ and $\lambda_c = 2$. As E-MPC

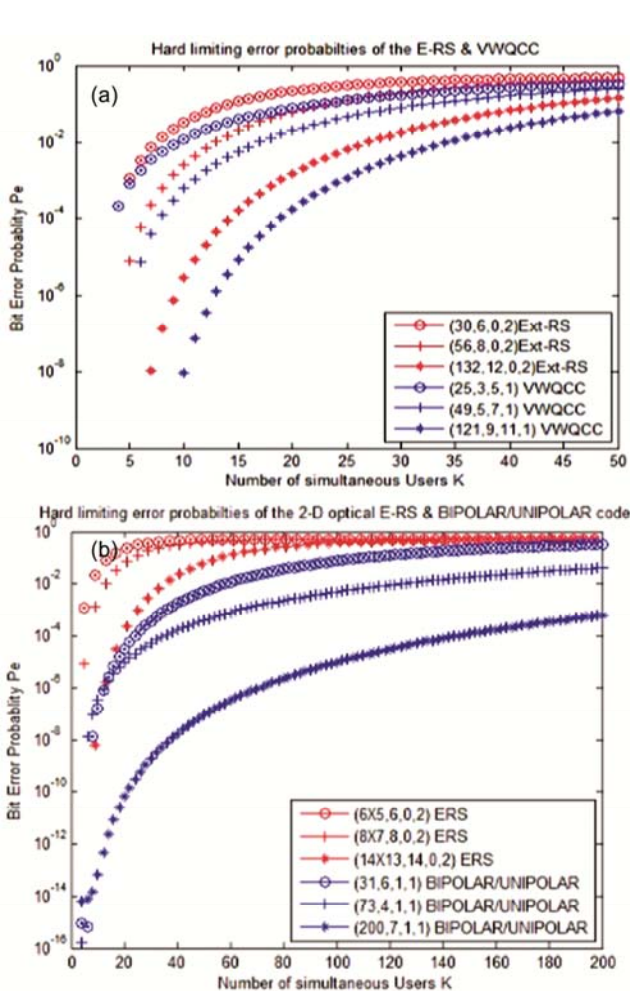


Fig. 2 — (a) E-RS vs. VWQCC (b) E-RS vs Bipolar/Unipolar

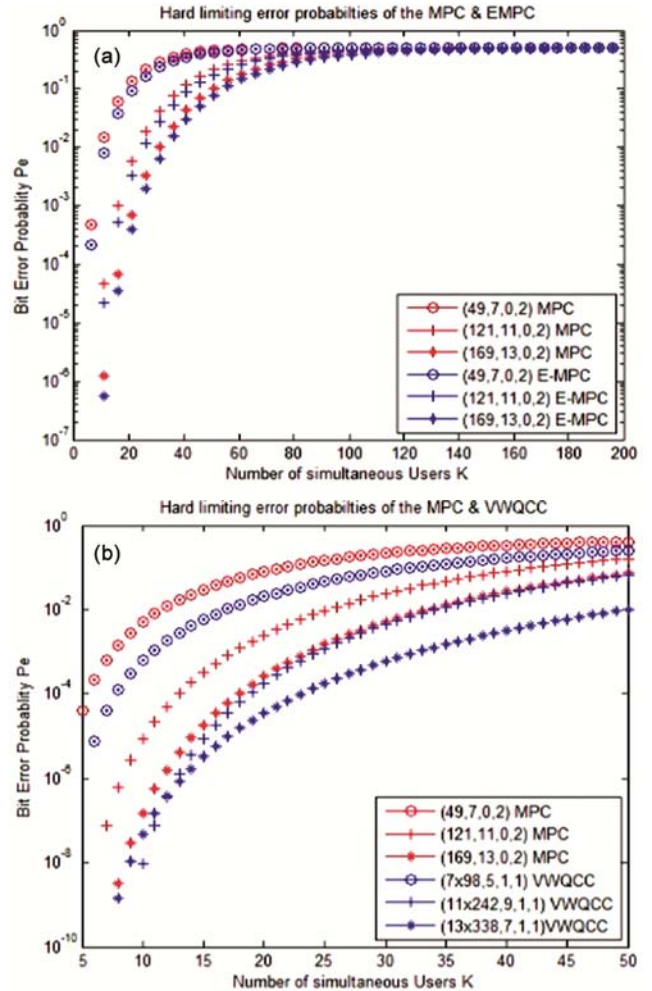


Fig. 3 — (a) E-MPC vs. MPC (b) MPC vs. VWQCC

have optimal cardinality and accounts more number hit probability equations as compared to MPC which lacks these features hence E-MPC performs better than MPC. In Fig. 3 (b) VWQCC clearly outperforms MPC for same number of prime number $p = \{7, 11, 13\}$ and for same number of available wavelengths ($=p$). Due to all these properties VWQCC has better code performance than MPC.

MPC vs Bipolar/Unipolar and E-MPC vs. VWQCC

In Fig. 4(a) Bipolar/Unipolar clearly outperforms MPC, because Bipolar/Unipolar code possess greater LN product value and better correlation properties viz. auto-and cross-correlation values *i.e.* $\lambda_a = 1$ and $\lambda_c = 1$ for Bipolar/Unipolar whereas $\lambda_a = 0$ and $\lambda_c = 2$ for MPC. Therefore MPC despite having heavier code weight doesn't perform better than Bipolar/Unipolar. Figure 4(b) clearly shows VWQCC is superior in

performance to E-MPC. Although E-MPC has heavier code weight than VWQCC but VWQCC have some superior properties which is the cause of its better performance than that of E-MPC. VWQCC have larger code length $N = 2p^2$ ($N = p$ for E-RS) along with higher value of Auto-correlation $\lambda_a = p$ ($\lambda_a = 0$ for E-MPC) as well as Low value of Cross-correlation $\lambda_c = 1$ which eventually cause low BER.

E-MPC and VWQCC vs Bipolar/Unipolar

In Fig. 5(a), Bipolar/Unipolar clearly outperforms E-MPC, because Bipolar/Unipolar code possess greater LN product value and better correlation properties viz. auto-and cross-correlation values *i.e.* $\lambda_a = 1$ and $\lambda_c = 1$ for Bipolar/Unipolar whereas $\lambda_a = 0$ and $\lambda_c = 2$ for E-MPC. Therefore E-MPC despite having heavier code weight doesn't perform better than Bipolar/Unipolar. Figure 5(b) clearly shows Bipolar/Unipolar are superior in performance to VWQCC. Bipolar/Unipolar has more number of

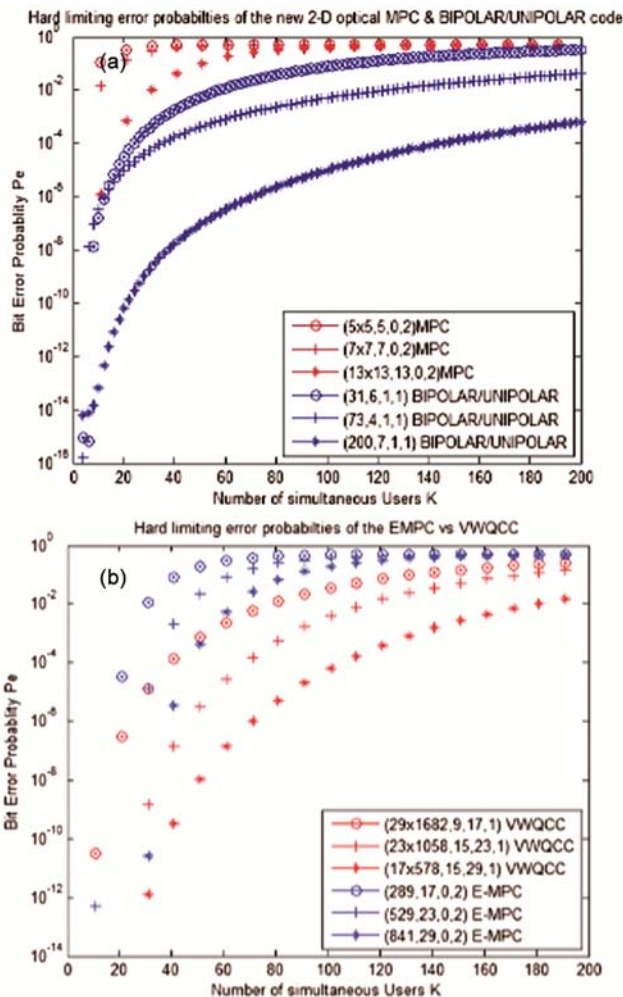


Fig. 4 — (a) MPC vs. Bipolar/Unipolar (b) EMPC vs. VWQCC

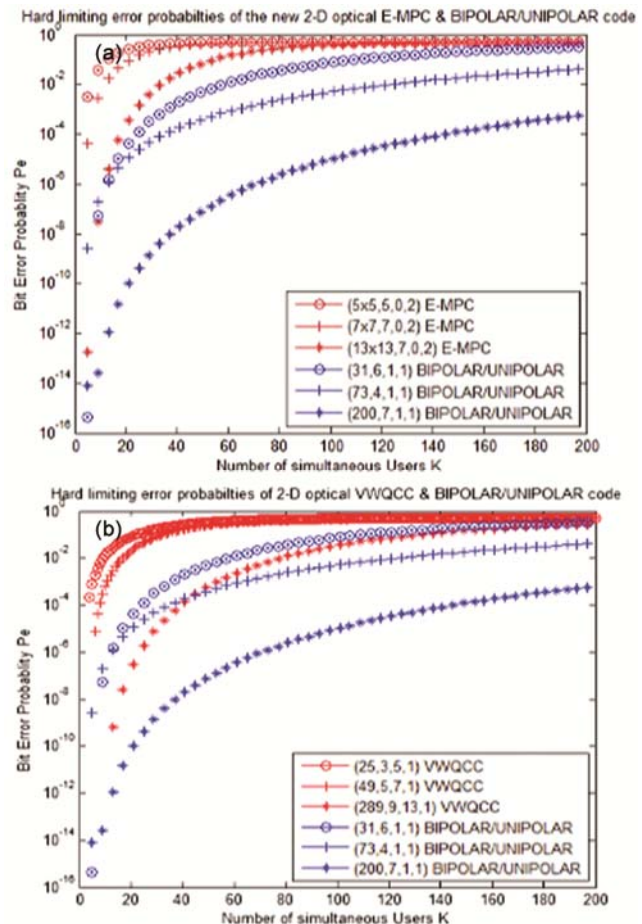


Fig. 5 — (a) EMPC vs. Bipolar/Unipolar (b) VWQCC vs. Bipolar/Unipolar

wavelength than VWQCC also Bipolar/Unipolar code have some superior properties which is the cause of its better performance than that of VWQCC. VWQCC have higher value of Auto-correlation $\lambda_a = p$ ($\lambda_a = 1$ for Bipolar/Unipolar) but have equal value of Cross-correlation $\lambda_c = 1$ with that of Bipolar/Unipolar which eventually cause less number of hits and hence low BER but high LN product value of Bipolar/Unipolar causes better performance than VWQCC as more available wavelengths are there for data transmission.

4 Conclusions

In this paper, five distinct optical codes E-RS, MPC, E-MPC, VWQCC and Bipolar/Unipolar belonging to same family of algebraic constructed codes are analyzed and investigated for the best promising code to work for optical CDMA in practical environment including multiple access interference. These codes are compared by taking hard limiting error probability as performance metrics. Investigations revealed that Bipolar/Unipolar code of

this family outperforms all other codes on the basis of HEP including multiple access interference for simultaneous number of users. Bipolar/Unipolar codes provide better efficiency in comparison with all other codes due to their inherent good correlation properties, higher user capacity and availability of large number of wavelengths for usage. Hence Bipolar/Unipolar code is the best candidate for providing best system performance in presence of MUI for optical CDMA.

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