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Parabola instability of Rayleigh-Taylor instability in quantum magnetized plasmas

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Quantum effects on the Rayleigh-Taylor instability in an inhomogeneous stratified incompressible plasma layer have been investigated. The combined effect of horizontal and vertical magnetic field has been taken into account. The solution of the linearized equations of the system together with the boundary conditions leads to derive the dispersion relation. The behaviour of growth rates with respect to the quantum effects beside the combined effect of magnetic field components has been analyzed. The results show that the growth rate depends on the $\lambda^*=\lambda L_D$ (λ and L_D are constants). The square normalized growth rate as a function λ^* describes in a parabola equation formula, where the maximum instability happens at $\lambda^*=-0.5$.

Keywords: Parabola instability, Quantum magnetized plasmas, Rayleigh-Taylor instability

1 Introduction

In recent years, there has been a considerable interest to study the quantum plasmas, where quantum plasmas have a wide range of applications. For example, quantum plasmas play an important role in ultra-small electronic devices[1], dense astrophysical plasmas system[2,3], intense laser-matter experiments[4] and nonlinear quantum optics[5,6]. Quantum effects become important to study the behaviour of charged plasma particles when the de Broglie wavelength of charge carriers becomes equal to or greater than the dimension of the quantum plasma system[7,8]. Quantum plasmas can be composed of electrons, ions, positrons, holes and/or grains. Two models are used to study quantum plasmas systems. The first one is the Wigner-Poisson and the other is the Schrödinger-Poisson approaches[7]. Two models are widely used to describe the statistical and hydrodynamic behaviour of the plasma particles at quantum scales in quantum plasmas. The quantum hydrodynamic model was introduced in semiconductor physics to describe the transport of charge, momentum and energy in plasma[9]. Different models in plasmas have been studied to clear the role of quantum corrections. For example, Haas *et al*[10]. studied a quantum multi-stream model for one and two stream plasmas instabilities. By employing the Wigner-Poisson model Bengt *et al*[11]. studied the dispersion properties of electrostatic oscillations in quantum plasmas for different parameters ranging from semiconductor plasmas to typical metallic electron densities and densities corresponding to compressed matter and dense astrophysical objects. Haas[12] extended the QHD equations for quantum magneto-plasmas and presented QMHD model by using the Wigner-Maxwell equations. The quantum effects on the internal waves of RTI in plasma have been studied by Bychkov *et al*[13]. The effect of quantum term on RTI in the presence of horizontal magnetic field has been studied by Jintao *et al*[14]. Hoshoudy[15] studied the same model in the presence of vertical magnetic field. RTI in quantum plasmas with *para*- and ferromagnetic properties has been studied by Modestov *et al*[16]. RTI in a non-uniform dense quantum magneto-plasma has been studied by Ali *et al*[17]. The role of quantum term on RTI through porous media has been studied by Hoshoudy[18,19].. The effect of quantum correction with streaming on linear and non-linear properties of electron plasma waves using the QHD model in unmagnetized, collisionless, ultracold electron-ion quantum plasma with streaming motion has been studied by Swarniv *et al*[20].

In the present paper, the combined effect of horizontal and vertical magnetic field on RTI of stratified plasmas layer with quantum effects has been considered. The normalized growth rate as a function of the physical parameters of the problem has been derived and examined.

2 Linearized Perturbation Equations

fluid of electrons and immobile ions has been considered. The plasma has been immersed in a magnetic field $\vec{B}$, where the relevant linear perturbation equations may be written as[14-19]:

$$\rho_0 \frac{\partial \vec{U}_1}{\partial t} = \vec{\nabla} P_1 + \rho_1 \vec{g} + \frac{1}{\mu_0}[(\nabla \times B_0)\times B_1 + (\nabla \times B_1)\times B_0] + \vec{Q}_1 \quad \ldots(1)$$

$$\nabla \cdot \vec{U}_1 = 0 \quad \ldots(2)$$

$$\frac{\partial \mathrm{B}_1}{\partial t} = \vec{\nabla}(\vec{\mathrm{U}}_1 \times \vec{\mathrm{B}}_0) \quad \ldots(3)$$

$$\frac{\partial \rho_1}{\partial t} + (\vec{\mathrm{U}}_1 \cdot \vec{\nabla})\rho_0 = 0 \quad \ldots(4)$$

where $\vec{\mathrm{U}}_1$, p_1, B_1, ρ_1 and Q_1 are the perturbations in the velocity $\vec{\mathrm{U}}$, pressure p, magnetic field $\vec{\mathrm{B}}$, density ρ and the quantum tem $\vec{\mathrm{Q}}=\frac{\hbar^2}{2m_e m_i}\rho\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$, where $\hbar$ is the Planck's constant, m_e the electron mass, m_i the ion mass and Q_1 is given in the formula:

$$Q_1 = \frac{\hbar^2}{2m_e m_i}\left\{\begin{array}{l} \frac{1}{2}\nabla(\nabla^2\rho_1) - \frac{1}{2\rho_0}\nabla\rho_1\nabla^2\rho_0 \\ -\frac{1}{2\rho_0}\nabla\rho_0\nabla^2\rho_1 + \frac{\rho_1}{2\rho_0^2}\nabla\rho_0\nabla^2\rho_0 \\ -\frac{1}{2\rho_0}\nabla(\nabla\rho_0.\nabla\rho_1) + \frac{\rho_1}{4\rho_0^2}\nabla(\nabla\rho_0)^2 \\ +\frac{1}{2\rho_0^2}(\nabla\rho_0)^2\nabla\rho_1 + \frac{1}{\rho_0^2}(\nabla\rho_0.\nabla\rho_1)\nabla\rho_0 \\ -\frac{\rho_1}{\rho_0^3}(\nabla\rho_0)^3 \end{array}\right\}$$

Now $\vec{U}_1 = (u_{x1}, u_{y1}, u_{z1})$, $\vec{\mathrm{B}}_1 = (B_{x1}, B_{y1}, B_{z1})$, $\vec{\mathrm{g}}=(0,0,-g)$, $\rho_0 = \rho_0(z), p_0 = p_0(z)$, $\frac{dp_0}{dz} = -\rho_0 g$ and $\vec{\mathrm{B}}_0 = B_{x0}(z)\,\vec{e}_x + B_{z0}(z)\,\vec{e}_z$ Also, it is assumed that the perturbation in all physical quantity takes the form:

$$\psi_1(x,y,z,t) = \psi_1(z)\exp\{i\,(k_x\,x + k_y\,y - \omega t)\}$$

where k_x and k_y are horizontal components of the wave-number vector $\vec{k}$ ($k^2 = k_x^2 + k_y^2$) and ω (may be complex ($\omega = \omega_r + i\gamma$)) is the frequency of perturbations or the rate at which the system departs from equilibrium thee initial state. Then, the system of Eqs (1)-(4) are:

$$-i\rho_0\omega u_{x1} = -ik_x p_1 + \frac{1}{\mu_0}\left\{B_{z1}\frac{\partial B_{x0}(z)}{\partial z} + B_{z0}(z)\left[\frac{\partial B_{z1}(z)}{\partial z} - ik_x B_{z1}\right]\right\} + \bar{Q}_{x1} \quad (5)$$

$$-i\rho_0\omega u_{y1} = -ik_y p_1 + \frac{1}{\mu_0}\left\{\begin{array}{l} iB_{x0}(z)[k_x B_{y1} - k_y B_{x1}] \\ +B_{z0}(z)\left[\frac{\partial B_{y1}(z)}{\partial z} - ik_y B_{z1}\right]\end{array}\right\} + \bar{Q}_{y1} \quad \ldots(6)$$

$$-i\rho_0\omega u_{z1} = -\frac{\partial p_1}{\partial z} - \rho_1 g + \frac{1}{\mu_0}\left\{\begin{array}{l} -B_{x1}\frac{\partial B_{x0}(z)}{\partial z} \\ +B_{x0}(z)\left[ik_x B_{z1} - \frac{\partial B_{x1}(z)}{\partial z}\right]\end{array}\right\} + \bar{Q}_{z1} \quad \ldots(7)$$

$$ik_x u_{x1} + ik_y u_{y1} + \frac{\partial u_{z1}}{\partial z} = 0 \quad \ldots(8)$$

$$-i\omega\{B_{x1}, B_{y1}, B_{z1}\} = \left\{\begin{array}{l} \left[\begin{array}{l} -ik_y\,\mathrm{B}_{x0}(z)u_{y1} + \frac{\partial}{\partial z}(B_{z0}(z)u_{x1} \\ \qquad - B_{x0}(z)u_{z1}) \end{array}\right] \\ \left[ik_x B_{z0}(z)u_{y1} - \frac{\partial}{\partial z}[B_{z0}(z)u_{y1}]\right] \\ [ik_x(B_{x0}(z)u_{z1} - B_{z0}(z)u_{x1}) \\ \quad - ik_y\,B_{z0}(z)u_{y1}] \end{array}\right\} \quad \ldots(9)$$

$$-i\omega\rho_1 + u_{z1}\frac{d\rho_0}{dz} = 0, \quad \ldots(10)$$

The terms $\bar{Q}_{x1}$, $\bar{Q}_{y1}$, $\bar{Q}_{z1}$ are referred to Ref. 18.

Eliminating some variables from the system of Eqs (5)-(10), we get a differential equation in u_{z1}

$$\frac{B_{z0}^2(z)}{\mu_0}\frac{d^4u_{z1}}{dz^4}+\frac{1}{\mu_0}\left\{\begin{matrix}4B_{0z}(z)\left(\frac{dB_0(z)}{dz}\right)\\ +2ik_xB_{x0}(z)B_{z0}(z)\end{matrix}\right\}\frac{d^3u_{z1}}{dz^3}$$
$$+\{\rho_0\omega^2+k^2A+\bar{A}\}\frac{d^2u_{z1}}{dz^2}+\left\{\omega^2\left(\frac{d\rho_0}{dz}\right)+k^2B+\bar{B}\right\}$$
$$\times\frac{du_{z1}}{dz}-k^2\left\{\rho_0\,\omega^2-(C-g)\left(\frac{d\rho_0}{dz}\right)-\bar{C}\right\}u_{z1}=0 \quad \ldots(11)$$

where A, B, C are given in Eq. (27), Ref. 18, while

$$\bar{A}=\frac{1}{\mu_0}\left\{-k_x^2B_{x0}^2(z)+3B_{z0}(z)\frac{d^2B_{z0}(z)}{dz^2}+2\left(\frac{dB_{z0}(z)}{dz}\right)^2\right.$$
$$\left.-k^2B_{z0}^2(z)+3ik_x\left[B_{x0}(z)\frac{dB_{z0}(z)}{dz}+B_{z0}(z)\frac{dB_{x0}(z)}{dz}\right]\right\}$$
$$\bar{B}=\frac{1}{\mu_0}\left\{-2k_x^2B_{x0}(z)\frac{dB_{x0}(z)}{dz}+B_{z0}(z)\frac{d^3B_{z0}(z)}{dz^3}\right.$$
$$+\frac{dB_{z0}(z)}{dz}\frac{d^2B_{z0}(z)}{dz^2}-2k^2B_{z0}(z)\frac{dB_{z0}(z)}{dz}$$
$$+ik_x\left[2B_{z0}(z)\frac{d^2B_{x0}(z)}{dz^2}-2k^2B_{x0}(z)B_{z0}(z)\right.$$
$$\left.\left.+2\frac{dB_{x0}(z)}{dz}\frac{dB_{z0}(z)}{dz}+\frac{dB_{x0}(z)}{dz}\frac{d^2B_{z0}(z)}{dz^2}\right]\right\}$$
$$\bar{c}=\frac{1}{\mu_0}\left\{k_x^2B_{x0}^2(z)+ik_x\left[\frac{1}{k^2}\left(B_{z0}(z)\frac{d^3B_{x0}(z)}{dz^3}\right.\right.\right.$$
$$\left.+\frac{dB_{z0}(z)}{dz}\frac{d^3B_{x0}(z)}{dz^3}\right)-B_{x0}(z)\frac{dB_{z0}(z)}{dz}$$
$$\left.\left.-B_{z0}(z)\frac{dB_{x0}(z)}{dz}\right]\right\} \quad \ldots(12)$$

3 Continuously Stratified Plasma

For the case of incompressible continuously stratified quantum plasma layer of thickness h units confined between two rigid boundaries, in which the density and magnetic field distribution are given, respectively, by:

$\rho_0(z)=\rho_0(0)\exp(z/L_D)$,
$B_{x0}(z)=B_{x0}(0)\exp(z/2L_D)$,
$B_{z0}(z)=B_{z0}(0)\exp(z/2L_D)$ and
$\rho_0(0), B_{x0}(0), B_{z0}(0), L_D$
are constants, then Eq. (12) takes the form:

$$\upsilon_{f_z}^2\frac{d^4u_{z1}}{dz^4}+\frac{2}{L_D}\left\{\upsilon_{f_z}^2+ik_xL_D\upsilon_{f_z}\upsilon_{f_x}\right\}\frac{d^3u_{z1}}{dz^3}$$
$$+\left\{\omega^2-k_x^2\upsilon_{f_x}^2+\upsilon_{f_z}^2\left(\frac{5}{4L_D^2}-k^2\right)+\frac{3ik_x\upsilon_{f_z}\upsilon_{f_x}}{L_D}-\omega_q^2\right\}$$
$$\times\frac{d^2u_{z1}}{dz^2}+\frac{1}{L_D}\left\{\omega^2-k_x^2\upsilon_{f_x}^2+\upsilon_{f_z}^2\left(\frac{1}{4L_D^2}-k^2\right)+ik_xL_D\upsilon_{f_z}\upsilon_{f_x}\right.$$
$$\left.\times\left(\frac{5}{4L_D^2}-2k^2\right)-\omega_q^2\right\}\frac{du_{z1}}{dz}-k^2\left\{\omega^2+\frac{g}{L_D}-k_x^2\upsilon_{f_x}^2\right.$$
$$\left.-ik_x\upsilon_{f_z}\upsilon_{f_x}\left(\frac{1}{4k^2L_D^3}-\frac{1}{L_D}\right)-\omega_q^2\right\}u_{z1}=0 \quad \ldots(13)$$

where $\omega_q^2=\dfrac{\hbar^2k^2}{4L_D^2m_em_i}$ represents quantum effect, $\upsilon_{f_x}^2=\dfrac{B_{x0}^2(0)}{\mu_0\rho_0(0)}$ and $\upsilon_{f_z}^2=\dfrac{B_{z0}^2(0)}{\mu_0\rho_0(0)}$ are Alfvén velocity. Since at the boundaries, the normal component of the velocity must vanish, therefore, we have $u_{z1}=0$ at $z=0$, $z=h$. Then, we can put the solution of Eq. (13) in the form $u_{z1}=\sin\left(\dfrac{n\pi}{h}z\right)\exp(\lambda z)$ and substituting into Eq. (13), the dispersion relation may be written as :

$$4\lambda\upsilon_{f_z}^2\left\{\lambda^2-\left(\frac{n\pi}{h}\right)^2\right\}+2\lambda\left\{\begin{matrix}\omega^2-k_x^2\upsilon_{f_x}^2\\ +\upsilon_{f_z}^2\left(\frac{5}{4L_D^2}-k^2\right)-\omega_q^2\end{matrix}\right\}$$
$$+\frac{2\upsilon_{f_z}^2}{L_D}\left\{3\lambda^2-\left(\frac{n\pi}{h}\right)^2\right\}+\frac{1}{L_D}\left\{\begin{matrix}\omega^2-k_x^2\upsilon_{f_x}^2\\ +\upsilon_{f_z}^2\left(\frac{1}{4L_D^2}-k^2\right)-\omega_q^2\end{matrix}\right\}$$
$$+ik_x\upsilon_{f_z}\upsilon_{f_x}\left\{\frac{6\lambda}{L_D}+2\left(3\lambda^2-\left(\frac{n\pi}{h}\right)^2\right)+\left(\frac{5}{4L_D^2}-2k^2\right)\right\}=0 \quad \ldots(14)$$

$$\upsilon_{f_z}^2\left\{\lambda^4+\left(\frac{n\pi}{h}\right)^4-6\lambda^2\left(\frac{n\pi}{h}\right)^2\right\}+\frac{2\lambda}{L_D}\{\upsilon_{f_z}^2+ik_xL_D\upsilon_{f_z}\upsilon_{f_x}\}$$
$$\times\left\{\lambda^2-3\left(\frac{n\pi}{h}\right)^2\right\}+\left\{\omega^2-k_x^2\upsilon_{f_x}^2+\upsilon_{f_z}^2\left(\frac{5}{4L_D^2}-k^2\right)\right.$$

$$+\frac{3ik_x\upsilon_{f_z}\upsilon_{f_x}}{L_D}-\omega_q^2\Bigg\}\left\{\lambda^2-\left(\frac{n\pi}{h}\right)^2\right\}$$
$$+\frac{\lambda}{L_D}\left\{\omega^2-k_x^2\upsilon_{f_x}^2+\upsilon_{f_z}^2\left(\frac{1}{4L_D^2}-k^2\right)\right.$$
$$\left.+ik_xL_D\upsilon_{f_z}\upsilon_{f_x}\left(\frac{5}{4L_D^2}-2k^2\right)-\omega_q^2\right\}$$
$$-k^2\begin{Bmatrix}\omega^2+\dfrac{g}{L_D}-k_x^2\upsilon_{f_x}^2\\-ik_x\upsilon_{f_z}\upsilon_{f_x}\left(\dfrac{1}{4k^2L_D^3}-\dfrac{1}{L_D}\right)-\omega_q^2\end{Bmatrix}=0 \qquad \ldots(15)$$

Now, we define the dimensionless quantities:

$$\omega^{*2}=\frac{\omega^2}{\omega_{pe}^2},\ \omega_q^{*^2}=\frac{\hbar^2}{4L_D^4m_em_i\omega_{pe}^2},\ \omega_{f_x}^{*2}=\frac{\upsilon_{f_x}^{*2}}{\omega_{pe}^2L_D^2}$$
$$\omega_{f_z}^{*^2}=\frac{\upsilon_{f_z}^{*2}}{\omega_{pe}^2L_D^2},\ \lambda^{*2}=\lambda^2L_D^2,\ h^{*2}=\frac{h^2}{L_D^2},\ k^{*^2}=k^2L_D^2$$
$$g^*=\frac{g}{\omega_{pe}^2L_D},\ \omega_{pe}=\left(\frac{\rho_0e^2}{m_e^2\varepsilon_0}\right)^{1/2} \qquad \ldots(16)$$

Then, Eqs (14) and (15), take the form, respectively

$$\{2\lambda^*+1\}\left\{(\omega^{*2}-k_x^{*2}\omega_{f_x}^{*2}-k^{*2}\omega_q^{*2})+\left[2\lambda^{*2}+2\lambda^*+\frac{1}{4}\right.\right.$$
$$\left.\left.-2\left(\frac{n\pi}{h^{*2}}\right)^2-k^{*2}\right]\right\}\omega_{f_z}^{*^2}+\left\{6\lambda^{*2}+6\lambda^*-2\left(\frac{n\pi}{h^{*2}}\right)^2\right.$$
$$\left.+\left(\frac{5}{4}-2k^{*2}\right)\right\}ik_x^*\omega_{f_x}^*\omega_{f_z}^*=0 \qquad \ldots(17)$$

$$\left\{\lambda^{*2}+\lambda^*-\left(\frac{n\pi}{h^*}\right)^2-k^{*2}\right\}\left\{\omega^{*2}-k_x^{*2}\omega_{f_x}^{*2}-k^{*2}\omega_q^{*2}\right\}$$
$$+\left\{\lambda^{*4}+\left(\frac{n\pi}{h^*}\right)^4-6\lambda^{*2}\left(\frac{n\pi}{h^*}\right)^2+2\lambda^*\left(\lambda^{*2}-3\left(\frac{n\pi}{h^*}\right)^2\right)\right.$$
$$\left.+\left(\frac{5}{4}-k^{*2}\right)\left(\lambda^{*2}-\left(\frac{n\pi}{h^*}\right)^2\right)+\lambda^*\left(\frac{1}{4}-k^{*2}\right)\right\}\omega_{f_z}^{*2}$$
$$+\left\{2\lambda^*\left(\lambda^{*2}-3\left(\frac{n\pi}{h^*}\right)^2\right)+3\left(\lambda^{*2}-\left(\frac{n\pi}{h^*}\right)^2\right)\right.$$
$$\left.+\lambda^*\left(\frac{5}{4}-2k^{*2}\right)+\left(\frac{1}{4k^{*2}}-1\right)\right\}ik_x^*\omega_{f_x}^*\omega_{f_z}^*-k^{*2}g^*=0$$
$$\ldots(18)$$

At $\omega^*=\omega_r^*+i\gamma$ and for $\omega_r^*=0$ (stable oscillations i.e. γ is the imaginary part of ω, see Refs 14 and 21) then Eqs. (17 and 18) are:

$$\left\{2\lambda^*+1\right\}\left\{\left\{-\gamma^2-k_x^{*2}\omega_{f_x}^{*2}-k^{*2}\omega_q^{*2}\right\}+\left[2\lambda^{*2}+2\lambda^*+\frac{1}{4}\right.\right.$$
$$\left.\left.-2\left(\frac{n\pi}{h^*}\right)^2-k^{*2}\right]\right\}\omega_{f_z}^{*2}+\left\{6\lambda^{*2}+6\lambda^*-2\left(\frac{n\pi}{h^*}\right)^2\right.$$
$$\left.+\left(\frac{5}{4}-2k^{*2}\right)\right\}ik_x^*\omega_{f_x}^*\omega_{f_z}^*=0 \qquad \ldots(19)$$

$$\left\{\lambda^{*2}+\lambda^*-\left(k^{*2}+\left(\frac{n\pi}{h^*}\right)^2\right)\right\}\left\{-\gamma^2-k_x^{*2}\omega_{f_x}^{*2}-k^{*2}\omega_q^{*2}\right\}$$
$$+\left\{\lambda^{*4}+\left(\frac{n\pi}{h^*}\right)^4-6\lambda^{*2}\left(\frac{n\pi}{h^*}\right)^2+2\lambda^*\left(\lambda^{*2}-3\left(\frac{n\pi}{h^*}\right)^2\right)\right.$$
$$\left.+\left(\frac{5}{4}-k^{*2}\right)\left(\lambda^{*2}-\left(\frac{n\pi}{h^*}\right)^2\right)+\lambda^*\left(\frac{1}{4}-k^{*2}\right)\right\}\omega_{f_z}^{*2}$$
$$+\left\{2\lambda^*\left(\lambda^{*2}-3\left(\frac{n\pi}{h^*}\right)^2\right)+3\left(\lambda^{*2}-\left(\frac{n\pi}{h^*}\right)^2\right)\right.$$
$$\left.+\lambda^*\left(\frac{5}{4}-2k^{*2}\right)+\left(\frac{1}{4k^{*2}}-1\right)\right\}ik_x^*\omega_{f_x}^*\omega_{f_z}^*-k^{*2}g^*=0$$
$$\ldots(20)$$

Now, in the case of $\omega_q^{*2}=\omega_{F_x}^{*2}=\omega_{f_z}^{*2}=0$, from Eq. (19) we get $\lambda^*=-\frac{1}{2}$, and substituting in Eq. (20) we find that the square normalized growth rate given by:

$$\gamma^2_{\text{Goldston and Rutherford}}=\frac{k^{*2}g^*}{\frac{1}{4}+\left(\frac{n\pi}{h^*}\right)^2+k^{*2}} \qquad \ldots(21)$$

this case is considered by Goldston *et al*[21].. For the case of $\omega_q^{*2}\neq 0,\ \omega_{f_x}^{*2}\neq 0,\ \omega_{f_z}^{*2}=0,$ a second time, from Eq. (19) we get $\lambda^*=-\frac{1}{2}$, and substituting in Eq. (20), then the square normalized growth rate given by:

$$\gamma^2_{\text{horizont magnetic field and quantum correction}} = \frac{k^{*2} g^*}{\frac{1}{4} + \left(\frac{n\pi}{h^*}\right)^2 + k^{*2}} - k_x^{*2}\omega_{f_x}^{*2} - k^{*2}\omega_q^{*2} \quad \ldots(22)$$

see Eq. (29) in Cao *et al*[14]. For $\omega_q^{*2} \neq 0$, $\omega_{f_x}^{*2} = 0$, $\omega_{f_z}^{*2} \neq 0$, again, from Eq. (19) we get $\lambda^* = -0.5$ and substituting in Eq. (20) the square normalized growth rate is given as:

$$\gamma^2_{\text{vertical magnetic field and quantum correction}} = \frac{g^* k^{*2}}{\frac{1}{4} + \left(\frac{n\pi}{h^*}\right)^2 + k^{*2}} - \frac{\left\{\left(\frac{n\pi}{h^*}\right)^2 + \frac{1}{4}\right\}\left\{\left(\frac{n\pi}{h^*}\right)^2 + k^{*2}\right\}\omega_{f_z}^{*2}}{\frac{1}{4} + \left(\frac{n\pi}{h^*}\right)^2 + k^{*2}} \quad \ldots(23)$$

(see Eq. (27) in Hoshoudy work[15])

Now, for the general case, fortunately, if we eliminate the term $ik_x^*\omega_{f_x}^*\omega_{f_z}^*$ between Eqs (19) and (20). Then, the square normalized growth rate is given in the form

$$\gamma^2 = \frac{\zeta}{\xi} \quad \ldots(24)$$

where

$$\xi = \left\{\lambda^{*2} + \lambda^* - k^{*2} - \left(\frac{n\pi}{h^*}\right)^2\right\} \times \left\{6\lambda^{*2} + 6\lambda^* - 2k^{*2} - 2\left(\frac{n\pi}{h^{*2}}\right)^2 + \frac{5}{4}\right\}$$
$$-\{2\lambda^* + 1\}\left\{2\lambda^*\left[\lambda^{*2} - 3\left(\frac{n\pi}{h^*}\right)^2\right] + 3\left[\lambda^{*2} - \left(\frac{n\pi}{h^*}\right)^2\right] + \lambda^*\left(\frac{5}{4} - 2k^{*2}\right) + \frac{1}{4k^{*2}} - 1\right\} \quad \ldots(25)$$

$$\zeta = \left\{6\lambda^{*2} + 6\lambda^* - 2k^{*2} - 2\left(\frac{n\pi}{h^{*2}}\right)^2 + \frac{5}{4}\right\} \times \left\{\left[\lambda^{*2} + \lambda^* - k^{*2} - \left(\frac{n\pi}{h^*}\right)^2\right](-k^{*2}\omega_q^{*2} - k_x^{*2}\omega_{f_x}^{*2})\right.$$
$$+\left[\lambda^{*4} + \left(\frac{n\pi}{h^*}\right)^4 - 6\lambda^{*2}\left(\frac{n\pi}{h^*}\right)^2 + \lambda^*\left(2\lambda^{*2} - k^{*2} - 6\left(\frac{n\pi}{h^*}\right)^2 + \frac{1}{4}\right) + \left(\frac{5}{4} - k^{*2}\right)\left(\lambda^{*2} - \left(\frac{n\pi}{h^*}\right)^2\right)\right]\omega_{f_z}^{*2} - k^{*2}g^*\Bigg\}$$
$$-(2\lambda^* + 1)\left\{(-k^{*2}\omega_q^{*2} - k_x^{*2}\omega_{f_x}^{*2}) + \left[2\lambda^{*2} + 2\lambda^* - k^{*2} + \frac{1}{4} - 2\left(\frac{n\pi}{h^{*2}}\right)^2\right]\omega_{f_z}^{*2}\right\}\left\{\lambda^{*3} + 3\lambda^{*2} - 6\lambda^*\left(\frac{n\pi}{h^*}\right)^2 - 3\left(\frac{n\pi}{h^*}\right)^2 + \lambda^*\left(\frac{5}{4} - 2k^{*2}\right) + \frac{1}{4k^{*2}} - 1\right\} \quad \ldots(26)$$

4 Results and Conclusions

The square of normalized growth rate λ^2 in Eq. (21) is a function in the dimensionless quantities $\omega_{f_x}^*$, $\omega_{f_z}^*$, ω_q^*, k^* and λ^* ($\lambda^* = \lambda L_D$, where λ is constant and L_D is the density-scale length). The dimensionless quantities $\omega_{f_x}^*$, $\omega_{f_z}^*$ and ω_q^* are the parameters of problem that may take different values. While the constant λ^* is unknown in this case. So firstly, we will try to discuss the role of constant λ^* on the square of normalized growth rate γ^2 in our selected problem.

Figure 1(a) shows the role of λ^* (≤ -0.5), for example $\lambda^* = -1.5$, -1, -0.5. One can see that the magnitudes of γ^2 decrease with decreasing of λ^*. While Fig. 1(b) shows the role of λ^* (≥ -0.5), for example $\lambda^* = -0.5$, 0.5, 1, it clear that, the magnitudes of γ^2 decrease with increasing of λ^*

In Figs 2(a), the square normalized growth rate γ^2 is plotted against λ^*($-2 < \lambda^* < 1$) at $k^{*2} = 5$, 7, 10. For the values λ^* that is less than -0.5($\lambda^* < 0.5$), the magnitude of γ^2 decreases with decreasing of λ^*, while for the values λ^* that is greater than -0.5($\lambda^* < -0.5$) the magnitude of γ^2 decreases with the magnitude of λ^* increases. These implies in the presence of both vertical and horizontal magnetic field and quantum effects, the system will be more stable at $\lambda^* \neq -0.5$, where the maximum instability happens at $\lambda^* = -0{,}5$. The discontinuous values are shown in Fig. 2(a), occur as the graph of a parabola. This parabola is symmetric about the λ^*-axis, the parabola opens downward and the maximum point (vertex) is $(\lambda^*, \gamma^2_{\max})$.

In the next steps, we will try to prove that the discontinuous values are shown in Fig. 2(a), already represent in parabola equation. So, we consider the relation between γ^2 and λ^* describes as parabola equation in the form $\gamma^2_{k^{*2}}(\lambda^*) = c\{\lambda^* + 0.5\}^2 + \gamma^2_{\max}$. From Fig. 2(a), we can rewrite the previous equation for random values, for example $k^{*2} = 5$, $\lambda^* = -1$ as:

$$\gamma^{*^2}_{k^{*2}=5}(-1) = 0.87614 = c\{-1+0.5\}^2 + 1.09967$$

this tends to $c = -0.89412$. Therefore, the discontinuous values in Fig. 2 (a) at $k^{*2} = 5$ can be represented as a continuous values as :

$$\gamma^{*^2}_{k^{*2}=5}(\lambda^*) = -0.89412\{\lambda^* + 0.5\}^2 + 1.09967$$

with error 0.0003.

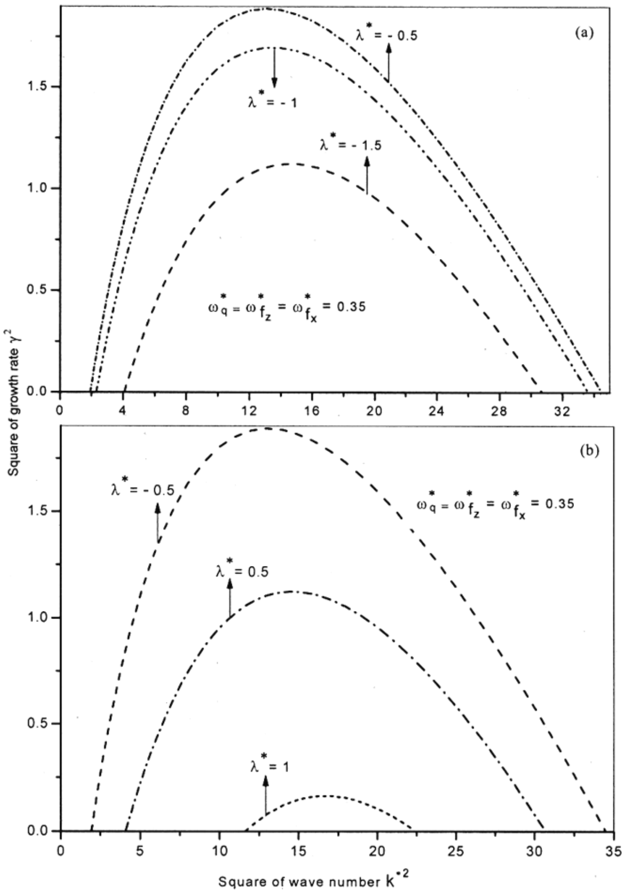

Fig. 1 — Square normalized growth rate (γ^2) against the square normalized wave number k^{*2} with different values of λ^* (a) at λ^*=−1.5, −1, −0.5 and (b) at λ^*=−0.5, 0.5, 1

Now, using the same way we can represent the discontinuous values in Fig.2 (a) at $k^{*2} = 7$, $k^{*2} = 10$, respectively, in the parabola equation formula as following :

$$\gamma^{*^2}_{k^{*2}=7}(\lambda^*) = -0.89375\{\lambda^* + 0.5\}^2 + 1.49464$$

with error 0.0003 and

$\gamma^{*^2}_{k^{*2}=10}(\lambda^*) = -0.85125\{\lambda^* + 0.5\}^2 + 1.8033$, with error 0.0007.

Comparing the values of the previous three equations at k^{*2}=5, 7, 10 and their counterpart in Fig. 2(a) are given in Fig. 2(b). One can see that the continuous values in the previous three equation (red circles) coincided with their counterpart (black circles) in Fig. 2(a). The same phenomenon can hold if we consider k^* is constant and the parameters $\omega^*_{f_x}$, $\omega^*_{f_z}$, ω^*_q take a different values. This implies that the relation between square normalized growth rate

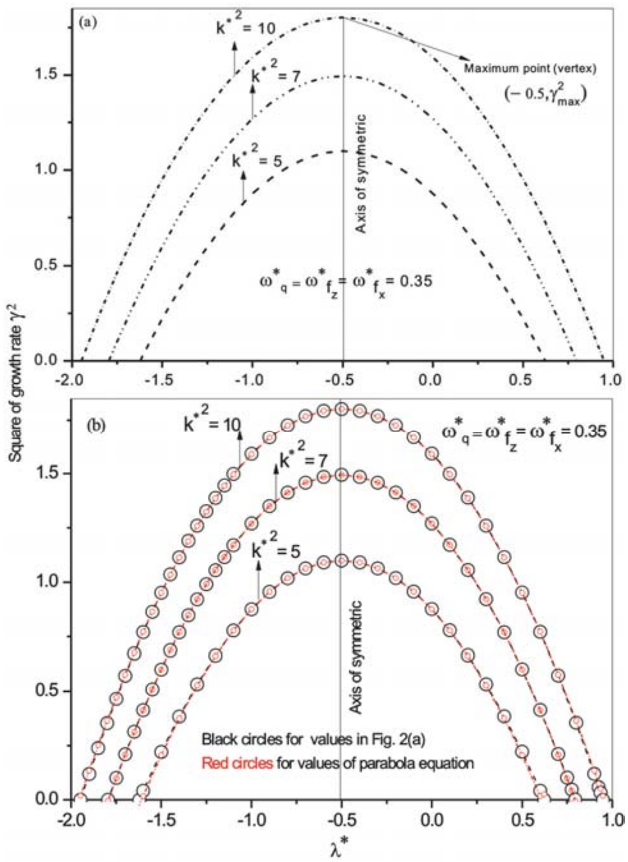

Fig. 2 — Square normalized growth rate (γ^2) as a function of the constant λ^* at $\omega^*_{f_x} = \omega^*_{f_z} = \omega^*_q = 0.35$, $k^{*2} = 5,7,10$ through the region λ^* $(-2 < \lambda^* < 1)$ (a) From Eq. (21) (b) The values (a) and their counterparts that generate by parabolas equations

γ^2 and λ^* [Eq. (21)] for any different values of the other parameters ($\omega^*_{f_x}$, $\omega^*_{f_z}$, ω^*_q and k^*) describe by parabola equation.

In the case of $\lambda^*=-0.5$ [the maximum point of instability at $\gamma^2(\lambda^*)$] and in Eq. (21), then the maximum square normalized growth rate $\gamma^2_{\max}$ gives as:

$$\gamma^2_{\max}(\lambda^*=-0.5)=\frac{k^{*2}g^*}{\frac{1}{4}+k^{*2}+\left(\frac{n\pi}{h^*}\right)^2}-\left\{k_x^{*2}\omega^{*2}_{f_x}+k^{*2}\omega^{*2}_x+\frac{\left[\left(\frac{n\pi}{h^*}\right)^4+\frac{1}{4}\left(\frac{n\pi}{h^*}\right)^2+\frac{1}{4}k^{*2}+\left(\frac{n\pi}{h^*}\right)^2k^{*2}\right]\omega^{*2}_{f_z}}{\frac{1}{4}+k^{*2}+\left(\frac{n\pi}{h^*}\right)^2}\right\} \quad \ldots(27)$$

The combined effects of horizontal, vertical magnetic field and quantum term on the considered system and that given in Eq. (21) (the general case) are shown in Fig. 3, where the square normalized growth rate γ^2 is plotted against the square normalized wave number k^{*2} at $\omega^*_{f_x}=\omega^*_{f_z},=\omega^*_q=0.35$ and $\lambda^*=-0.5$. Fig. 3(a) shows the effect of these parameters, where the values of γ^2 in the presence of these parameters, unaccompanied, are less than their magnitudes in the classical case. While the system will be more stable in the presence of these parameters together (see red solid curve). The increasing of these parameters together tends to be more stable than that occurs in Fig. 3(b), where the square normalized growth rate γ^2 has been given at $\omega^*_{f_x}=\omega^*_{f_z},=\omega^*_q=0.35$, =0.375, =0.4.

To conclude, the Rayleigh-Taylor instability in stratified plasma with combined effect of horizontal and vertical magnetic field components with quantum effects has been considered. The solution of the system leads to a dispersion relation. Some special cases are particularized to explain the previous roles that play the variables of the problem. Some stability diagrams are plotted and discussed. The results show that, as the growth rate depends the parameter's problem (horizontal, vertical components of magnetic field and quantum term) as well as depends on the parameter $\lambda^*=\lambda L_D$. Numerically, the maximum instability (maximum square normalized growth rate) happens at $\lambda^*=-0.5$ and analytically is given in Eq. (24). The system will be more stability for the values of λ^* that is different than -0.5. The square normalized growth rate as a function of λ^* Eq. (21) represents a parabola equation in the form $\gamma^2(\lambda^*)=c\{\lambda^*+0.5\}^2+\gamma^2_{\max}$, with different values of $\omega^*_{f_x}$, $\omega^*_{f_z}$, ω^*_q and k^*. Finally, our results indicate that the quantum mechanical effects are shown to suppress the RTI, that use, for example, in the celestial, astrophysics and space physics. Furthermore, it is shown that the self-generated magnetic field plays a more significant role in ICF experiments with quantum effects.

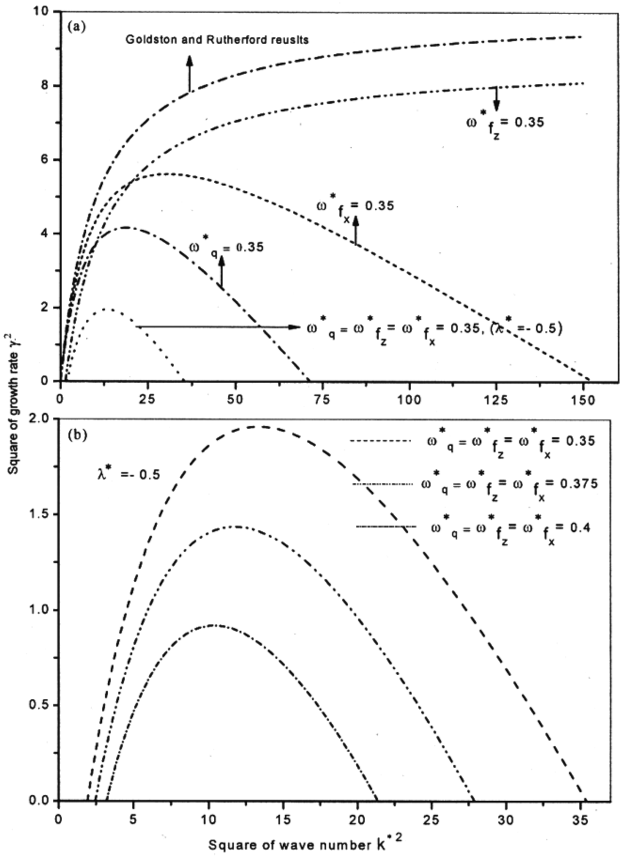

Fig. 3 — Square normalized growth rate (γ^2) against the square normalized wave number k^{*2} in the presence of $\omega^*_{f_x}$, $\omega^*_{f_z}$, ω^*_q (a) unaccompanied and huddled at $\omega^*_{f_x}=\omega^*_{f_z}=\omega^*_q=0.35$, (b) huddled at $\omega^*_{f_x}=\omega^*_{f_z}=\omega^*_q=0.35,\ 0.375,\ 0.4$

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