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Large band gaps in two-dimensional phononic crystals with Jerusalem cross slot structures

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The band gap properties in a novel two-dimensional phononic crystal with Jerusalem cross slot structures have been investigated theoretically in the present paper. The dispersion relations, the power transmission spectra and the displacement fields of the eigenmodes are calculated by using the finite element method. Numerical results show that the proposed structures with periodic Jerusalem cross slots can yield large band gaps in the low-frequency range as compared to the typical phononic crystals composed of periodic square rods embedded in a homogenous matrix. The formation mechanisms of the large low-frequency band gaps as well as the effects of the geometrical parameters on the band gaps are further explored numerically. Results show that the openings of the low-frequency band gaps are mainly attributed to the interaction between the local resonances of the square scatterers with Jerusalem cross slot structures and the traveling wave modes in the matrix. The band gaps can be significantly modulated by changing the geometrical parameters of Jerusalem cross slot structures.

Keywords: Band gaps, Phononic crystals, Jerusalem cross slot, Finite element method

1 Introduction

Over the last two decades, the propagation of elastic waves in periodic composite materials, known as phononic crystals (PCs), has attracted great interest because of their unique physical properties and potential applications[1-4]. With the existence of band gaps (BGs), phononic crystals possess extensive potential applications, such as vibration and noise reduction[5-7], sound filters[8,9] and waveguides[10,11]. Earlier studies have demonstrated that the formations of the BGs are mainly attributed to Bragg scattering and localized resonances, respectively. For the first mechanism, the BGs are attributed to the destructive interference between incident acoustic waves and reflections from the periodic scatterers. When wavelengths are of the order of the lattice constants, the phononic crystals[12-16] can yield complete BGs. For the second mechanism, the resonances of scattering units play a major role in the BGs which are dependent less on the periodicity and symmetry of the structure. The frequency range of the gaps could be almost two orders of magnitude lower than the usual Bragg gaps[17,18].

In order to promote the application of PCs in mechanical engineering, the acquisition of large and tunable BGs at low frequencies is of extremely importance. Caballero *et al*[19]. showed that absolute sonic band gaps produced by two-dimensional square and triangular lattices of rigid cylinders in air can be increased by reducing the structure symmetry. Li *et al*[20]. investigated the effects of orientations of square rods on the acoustic band gaps in two-dimensional periodic arrays of rigid solid rods embedded in air and concluded that the acoustic band gaps can be opened and enlarged greatly by increasing the rotation angle. Pennec *et al*[21]. and Wu *et al*[22]. investigated theoretically and experimentally the band structure of a phononic crystal of finite thickness constituted of a periodical array of cylindrical dots deposited on a thin plate of a homogeneous material and obtained a low-frequency gap, as compared to the acoustic wavelengths in the constituent materials, similarly to the case of locally resonant structures. Assouar *et al*[23]. introduced the concept of double-sided stubbed phononic plate and reported on the theoretical analysis of the enlargement of locally resonant acoustic band gap. Numerical results show that the enlargement of the relative bandwidth of the complete band gap is mainly due to the strong coupling between the same nature of resonant eigenmodes of stubs located in each plate side and the Lamb modes. Wang *et al*[24]. studied the band gap properties of two-dimensional phononic crystals with cross-like holes using the finite element method. Numerical results illustrated that systems of cross-like holes showed large band gaps at lower frequencies and the generation of the lowest band gap was the result of the local resonance of the periodically arranged lumps connected with narrow connectors. Yu *et al*[25]. studied the band gap properties of a two-dimensional phononic crystal with neck structure and showed that the proposed structures with necks can yield larger band gaps at lower frequencies. The openings of the band gaps were attributed to the resonance of the cylinders and the interaction between the cylinders and the matrix, resulting from the introduction of the necks.

In the present paper, the band gap properties in a novel two-dimensional phononic crystal with Jerusalem cross slot structures have been investigated theoretically. The dispersion relations and the power transmission spectra are calculated by using the finite element method (FEM), which has been proved in previous works to be an efficient method[23-25]. In contrast to the typical phononic crystals composed of periodic square rods embedded in a homogenous matrix, the proposed structures with periodic Jerusalem cross slots can yield large band gaps in the low-frequency range. The formation mechanisms of the low-frequency band gaps as well as the effects of the geometrical parameters on the band gaps are further explored numerically.

2 Model and Method of Calculation

The PC structure considered here is composed of periodic square rods embedded in a homogenous matrix with periodic Jerusalem cross slot structures. Figures 1(a) and (b) show the cross section of the proposed PC structure and its unit cell, respectively. The infinite system of the proposed PC structure is formed by repeating the unit cell periodically along the *x*- and *y*-directions simultaneously. The geometrical parameters of the Jerusalem cross slot structure are defined as follows: the parameters of the slot length are *l* and *m*, respectively, and the parameters of the slot width are *n* and *d*, respectively.

In order to investigate the BG characteristics of the proposed PC structure, the dispersion relations and the transmission spectra are calculated by using the FEM and Bloch theory. For the propagation of elastic waves in solid systems, the governing equations are given as follows:

$$\sum_{j=1}^{3}\left\{\frac{\partial}{\partial x_i}\left(\lambda\frac{\partial \boldsymbol{u}_j}{\partial x_j}\right)+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \boldsymbol{u}_i}{\partial x_j}+\frac{\partial \boldsymbol{u}_j}{\partial x_i}\right)\right]\right\} = \rho\frac{\partial^2 \boldsymbol{u}_i}{\partial t^2} \qquad i,j=x,y,z \qquad \ldots(1)$$

where ρ is the mass density and u is the displacement field. Since the infinite system is periodic along the

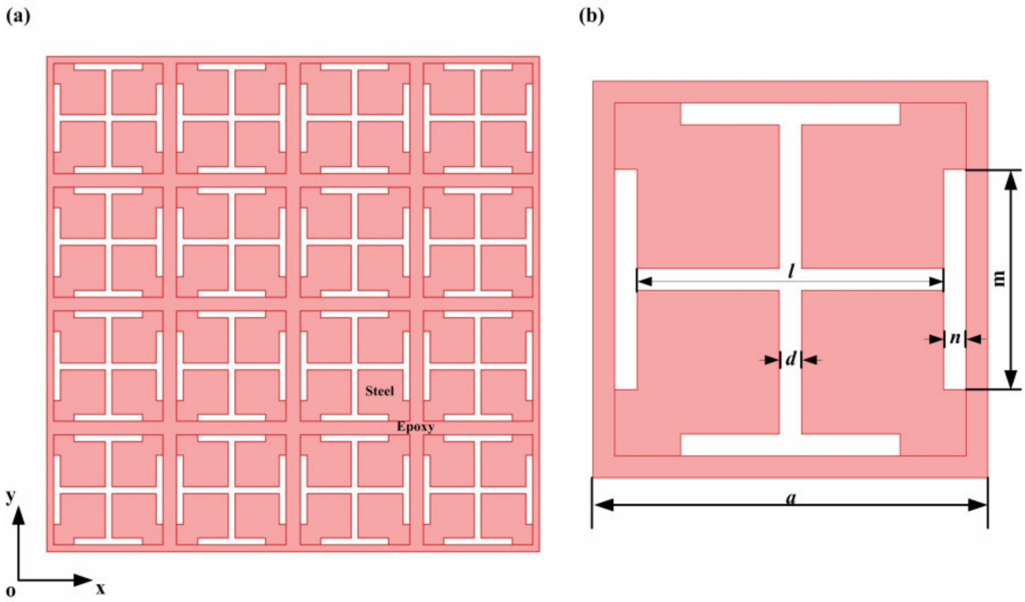

Fig. 1 — (a) 2D cross-section of the PC with Jerusalem cross slot structure, (b) Schematic diagram of the unit cell of the PC structure

x- and y- directions simultaneously, only the unit cell as shown in Fig. 1(b) needs to be considered. According to Bloch theorem, periodic boundary conditions are applied on the interfaces among all adjacent unit cells as follows:

$$u_i\left(x+a, y+a\right)=u_i\left(x, y\right)e^{j\left(k_x a+k_y a\right)} \quad i=x, y \qquad \ldots(2)$$

where k_x and k_y are the components of Bloch wave vector, respectively. With a given value of Bloch wave vector, a group of eigenvalues and eigenmodes can be calculated by solving the eigenvalue problem. Due to the periodicity of the PC in both x and y directions and the symmetry of the unit cell, the Bloch wave vector only needs to have a value along the border line of irreducible Brillouin zone. By varying the value of Bloch wave vector along the boundaries of the irreducible first Brillouin zone and solving the eigenvalue problem generated by the FEM algorithm and we can obtain the band structures (i.e., dispersion relations) of the proposed PC structures.

For the transmission spectrum, a finite array structure composed of N units in the x-direction is considered, while the periodic boundary conditions are still applied in the y-direction to represent the infinite units. The acceleration excitation source with single-frequency is incident from the left side of the finite array propagate along the x-direction. The transmitted acceleration value is detected and recorded on the right side of the structure. The transmission spectra are defined as the ratio of the transmitted power through the N layered finite system to the incident power. By varying the excitation frequency of the incident waves, the transmission spectra can be obtained.

3 Results and Discussion

3.1 Band Gaps of the PCs with Jerusalem Cross Slot Structures

In order to illustrate the band gap characteristics in the proposed PC structure, some numerical calculations are carried out. The material parameters are chosen as follows: C_{11}=264 GPa, C_{12}=102 GPa, C_{44}=81 GPa, and the density ρ=7850 kg/m^3 for steel; C_{11}=7.54 GPa, C_{12}=4.58 GPa, C_{44}=1.48 GPa, and the density ρ=1180 kg/m^3 for epoxy. The geometrical parameters are defined as follows: the lattice constant a = 36 mm, the slot length l = 28 mm and m=20 mm, the slot width n = 2 mm and d=2 mm. Figure 2(a) shows the dispersion relations for the infinite PC with periodic Jerusalem cross slot structures. It can be found that there are eight bands in the frequency range 0-15 kHz, where four complete band gaps (red regions in Fig. 2(a)) are involved. In order to further demonstrate the existence of the band gaps in the proposed PC structure, the transmission power spectra for a finite PC structure are calculated. The incident waves propagate along the x-direction through the eight rows of unit cells in the structure, and the transmission power spectra are defined as the ratio of the transmitted power through the eight-layered finite system to the incident power. Figure 2(b) shows the calculated transmission power spectrum. It can be observed that the transmission power spectrum is in reasonably consistence with the dispersion relations. The location and width of the complete band gaps also exhibit very good agreement between the band structure and the transmission power spectrum.

As a comparison, the band structure of the typical phononic crystals composed of periodic square steel rods embedded in a homogenous epoxy matrix without periodic Jerusalem cross slot structures has also been calculated. During the calculations, the material parameters and the lattice constant are still the same as those used in Fig. 2. Figure 3(a and b) shows the dispersion relations and the transmission power spectrum of the typical phononic crystal, respectively. One can observe that there is no complete band gap in the frequency range 0-15 kHz. Consequently, it can be concluded that the proposed PC structures with periodic Jerusalem cross slots can yield large band gaps in the low-frequency range, as

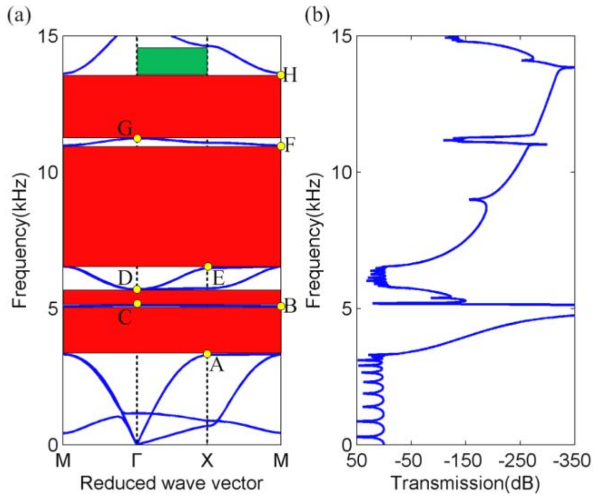

Fig. 2 — (a) Band structure of the proposed PC structure with a=36 mm, l = 28 mm, m=20 mm, n = 2 mm and d=2 mm, (b) transmission spectrum of the proposed PC structure

compared to the typical phononic crystals composed of periodic square rods embedded in a homogenous matrix.

In order to further intuitively illustrate the formation mechanism of the large band gaps in the low-frequency range, the displacement fields of eigenmodes at several edges of the bands marked in Fig. 2(a) are calculated as shown in Fig. 4. The modes on the lower and upper edges of the first two gaps have been investigated. It is observed that the total displacement fields of the corresponding point A are mainly embodied as the coupling between the torsion modes of the square scatterers with Jerusalem cross slot structures and the longitudinal vibration of the matrix along *x*-direction. Unlike mode A, modes B and C both belong to the fourth band, which is close to a flat band in the whole first BZ, and are actually the localized modes. The vibrations of modes B, C and D are mainly concentrated in the square scatterers

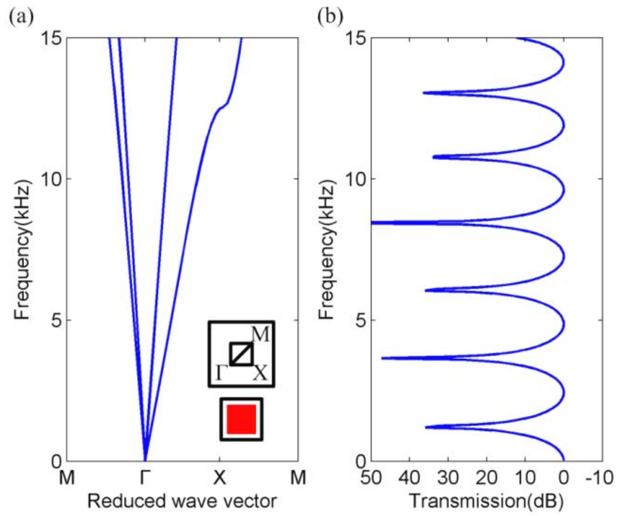

Fig. 3 — (a) Band structure of the typical PCs composed of periodic square steel rods embedded in a homogenous epoxy matrix without periodic Jerusalem cross slot structures, (b) transmission spectrum of the typical PC structure

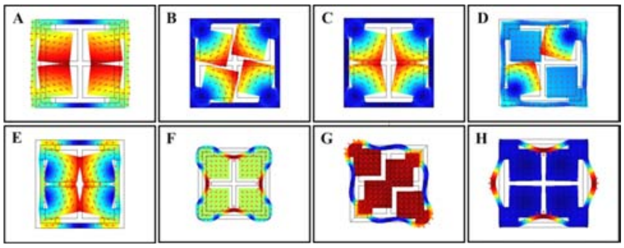

Fig. 4 — Total displacement fields of the corresponding point (*A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*) shown in Figure 2

with Jerusalem cross slot structures, while the matrix remains stationary. Then, the modes on the lower and upper edges of the third and fourth gaps have been studied. The vibration of mode E manifests the coupling between the torsion modes of the square scatterers and the longitudinal vibration of the matrix. However, the center of the torsion here is no longer the contact locations between the square scatterers and matrix. The total displacement fields of the corresponding points F and G are mainly the result of the interaction between the longitudinal vibration of the square scatterers along diagonal and the flexural vibrations of the matrix. In addition, it can be observed that the vibration of mode H is the localized flexural vibrations of the matrix, while the square scatterers remain stationary. The localized vibration modes possess limited capacity in wave propagation and therefore, lead to the attenuation of the transmission. Consequently, it can be concluded that the openings of the large low-frequency band gaps are mainly attributed to the interaction between the local resonances of the square scatterers with Jerusalem cross slot structures and the traveling wave modes in the matrix. In addition, because of the localization properties of modes B, C, F, and G, the first two gaps will be integrated into the first gap and the third and fourth gaps will be integrated into the second gap in the following analysis.

3.2. Effect of Geometry Parameters on the Band Gaps

To illustrate the effects of geometrical parameters on the band gaps, the calculation of band structures with one of the geometrical parameters changing, while the other geometrical parameters and material parameters are fixed during all the calculations, has been performed. Firstly, the effect of the slot width *d* on the band gaps has been investigated. Figure 5 shows the gap maps of the proposed PC structures as a function of the factor of the slot width *d*. One can observe that, with the increase of the slot width *d*, the lower and upper edges of both gaps move towards high-frequency region. For the first gap, the increase of the frequency of the upper edge is faster than that of the lower edge, resulting in the increase of the first gap width. In contrast, for the second gap, the increase of the frequency of the upper edge is slower than that of the lower edge, leading to the decrease of the second gap. It can be observed from Figs 2 and 4 that the formation mechanism of band gaps in the proposed PC structures is attributed to the interaction between the local resonances of the square scatterers

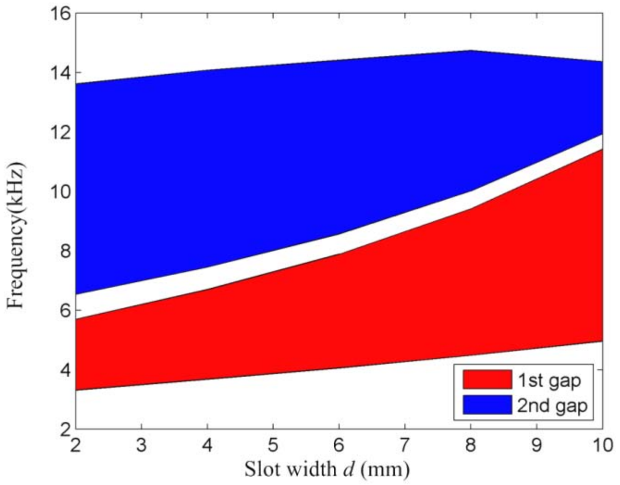

Fig. 5 — Gap maps of the proposed PC structures as a function of the slot width *d* with the lattice constant *a*=36 mm, the slot length *l* = 28 mm and *m*=20 mm, the slot width *n* = 2 mm

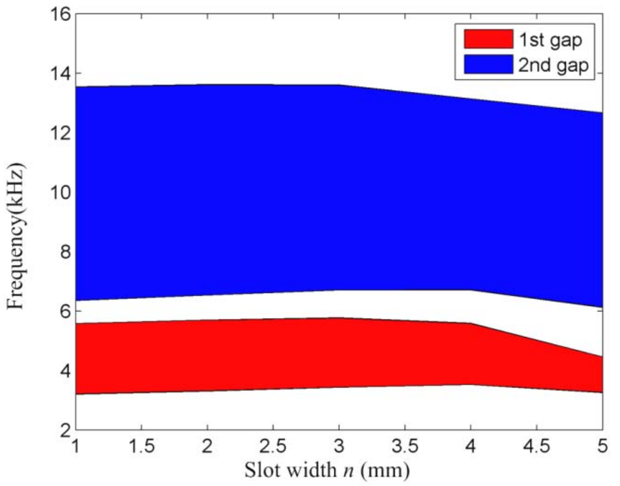

Fig. 6 — Gap maps of the proposed PC structures as a function of the slot width *n*, with the lattice constant *a*=36 mm, the slot length *l* = 28 mm and *m*=20 mm, the slot width *d* = 2 mm

with Jerusalem cross slot structures and the traveling wave modes in the matrix[24,25]. With the increase of the slot width *d*, the mass of the square scatterers decreases and the connection stiffness almost keeps unchanged, resulting in the increase of the local resonance frequency of the square scatterers.

Then, the effect of the slot width *n* on the band gaps has been studied numerically. Figure 6 shows the gap maps of the proposed PC structures as a function of the factor of the slot width *n*. It is seen that, with the increase of the slot width *n* from 1 to 5 mm, the lower and upper edges of both gaps as well as the gap width almost remain unchanged. As we know, with the increase of the slot width *n*, both the mass of the square scatterers and the connection stiffness decrease simultaneously, so the local resonance frequency of the square scatterers is almost unchanged.

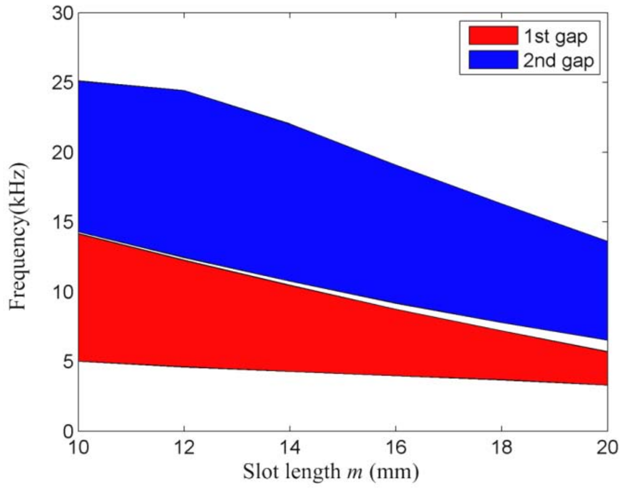

Fig. 7 — Gap maps of the proposed PC structures as a function of the slot width *m*, with the lattice constant *a*=36 mm, the slot length *l* = 28 mm, the slot width *n* = 2 mm and *d*=2 mm.

Finally, the effect of slot length *m* on the band gaps has been investigated. Figure 7 shows the gap maps of the proposed PC structures as a function of the factor of the slot length *m*. It can be observed that, with the increase of the slot length *m* from 10 to 20 mm, the lower and upper edges of both gaps are shifted to low-frequency region. Moreover, we can find that the decrease of the frequency of the upper edges is faster than that of the lower edge, resulting in the decrease of the both gaps width. With the increase of the slot length *m*, the mass of the square scatterers almost keeps unchanged, while the interface between the square scatterers and the matrix decreases, the constraint on the corrugation weakens and the connection stiffness decreases, resulting in the decrease of the local resonant eigenmodes of the square scatterers.

4 Conclusions

In this paper, the band gap properties in a novel two-dimensional phononic crystal with Jerusalem cross slot structures have been investigated by using the finite element method. In contrast to the typical phononic crystals composed of periodic square rods embedded in a homogenous matrix, the proposed structures with periodic Jerusalem cross slots can yield large band gaps in the low-frequency range. The displacement fields of the eigenmodes have been calculated to further intuitively illustrate the formation mechanism of the large band gaps in the low-frequency range. The openings of the low-frequency band gaps are mainly attributed to the coupling between the local resonances of the square scatterers with Jerusalem cross slot structures and the traveling wave modes in the matrix. Furthermore, the effects of the geometrical parameters on the band gaps are further explored numerically. The band gaps can be significantly modulated by changing the geometrical parameters of Jerusalem cross slot structures.

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