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Experimental scheme to precisely measure the magnetic penetration depth of a superconducting film

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The penetration depth is an important parameter of superconductors, but its absolute value is difficult to measure without a special device. By introducing the modern microfabrication technology, this paper proposes a new experimental scheme to measure the susceptibility of a superconducting film. By this new method, the absolute value of the penetration depth can be determined with satisfactory accuracy. The accuracy was estimated from the measurement error of the experimental scheme. This method should be useful for investigating the pairing mechanisms of superconductors by measuring the temperature dependence of the penetration depth.

Keywords: Magnetic penetration depth, Susceptibility, Superconducting film

1 Introduction

The London penetration depth λ of superconductors is directly related to the density n_s of the Cooper pairs $(\lambda^2 \propto 1/n_s)$. By measuring the temperature dependence of the penetration depth, we can better understand the pairing mechanism [1,2]. Therefore, the penetration depth has received much attention over many years, and has been measured by diverse methods, such as the muon-spin-relaxation technique[3], microwave surface impedance[4]and two-coil mutual inductance[5,6]. Many new types of superconductors have been founded in the past three decades, the temperature dependence of the penetration depth has become an important experimental evidence to judge the pairing mechanism of these superconductors.

However, since the penetration depth is very small (about 100 nm), its absolute value is difficult to measure with sufficient accuracy. Many experiments can only measure the relative values $\Delta\lambda$ of $\lambda(T)$, defined as $\Delta\lambda = \lambda(T) - \lambda(0)$ (where $\lambda(0)$ is the λ at $T = 0K$), rather than their absolute values. The absolute $\lambda(0)$ values are usually inferred by fitting the experimental $\Delta\lambda(T)$ values to a special theoretical model[7,8] of $\lambda(T)$. For some new superconductors that are not well- described by any existing theory, the theoretical model is decided by the researcher. Consequently, the absolute penetration depths and their temperature dependence depend on the researcher's interpretation, even when extracted from the same experimental data[7,9,10]. Clearly, this situation is unsatisfying, for a robust experiment should be independent of any theoretical model.

However, if we could accurately measure the absolute values of λ, the above uncertainty could be removed[7]. In the present paper, an improved experimental scheme for measuring the susceptibility of a superconducting film is proposed, by which the absolute values of $\lambda(T)$ can be precisely determined.

2 Difficulties in the Former Experiments

When a superconductor is placed in a magnetic field, its susceptibility approaches −1 if its size is much larger than its penetration depth λ. In contrast, the susceptibility of a superconducting film, whose thickness approximately equals λ, will appreciably differ from −1. Therefore, the penetration depth λ of a superconducting film can be deduced from its susceptibility values. According to the London equation, the susceptibility of a superconducting film with its surface parallel to the applied field[11] is given by:

$$\frac{\chi}{\chi_0} = 1 - \frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) \qquad \ldots(1)$$

where $\chi_0 = -1$ is the susceptibility of an infinitely thick plate (−1), and d is half the film thickness.

It is should be emphasized that Eq. (1) holds only when the film surface is exactly parallel to the magnetic field. Even a small angle between the film surface and the applied magnetic field induces a large magnetic moment perpendicular to the film. This moment will have a component along the axis of the measuring coils, unless that axis exactly aligns along the surface of the film. If the surface of the superconducting film is tilted by an angle θ from the magnetic field and an angle ϕ from the axis of the measuring coil, the measured susceptibility should be modified as follows:

$$\frac{\chi}{\chi_0}=\left(1+\frac{2l\theta\varphi}{3\pi Nd}\right)\left[1-\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right)\right] \qquad \ldots(2)$$

where l is the length of the superconducting film along the direction of the field, N is the number of disks that piled in a specimen, and the factor $\left(1+\frac{2l\theta\varphi}{3\pi Nd}\right)$ is a correcting factor denoted by α, i.e., $\alpha=\left(1+\frac{2l\theta\varphi}{3\pi Nd}\right)$.

Using the above method, Lock had measured the penetration depths of several superconducting films[11]. In this experiment, the superconducting films were evaporated on a (6 × 6) cm^2 mica sheet and divided into 45 pieces, which were then stacked and bound together to reduce the correcting factor α and increase the measurement signal. Since the film thickness was 250 nm, the correcting factor was $\alpha=(1+300\theta\varphi)$. To maintain small α, the angles θ and φ must be kept very small. For this purpose, measurements were performed in a small magnetic field under varying θ and φ. A specimen orientation was found for which the observed magnetic moment was insensitive to the direction of the applied field. At this orientation, the correcting factor should approximate unity. Obviously, such an experiment is difficult. Moreover, the absolute $\lambda(0)$ cannot be obtained by this approach, owing to irregularities in the surface of the film and slight lack of parallelism between adjacent sheets. For these reasons, this method was seldom adopted by researchers.

3 New Experimental Scheme

It is found that the above disadvantages can be eliminated if the method is modified using modern microfabrication technology. Furthermore, the modified method can precisely determine the absolute value of the penetration depth.

As revealed in the above analysis, the success of this experiment relies on minimizing the correcting factor α; that is, ensuring that $\frac{2l\theta\varphi}{3\pi Nd}<<1$. This may be achieved in two ways; increasing $3\pi Nd$ and reducing $2l\theta\varphi$.

A large value of $3\pi Nd$ requires that $N>>1$ or d is large. If $N>>1$, we must fabricate multiple samples, which increase the difficulty of the experiment. Moreover, adjacent sheets cannot be precisely parallel-aligned. Increasing the thickness $2d$ of the film can increase the magnetic moment of the sample and decrease the correcting factor α. But, if $d>>\lambda$, the measurement error in the absolute value of the penetration depth λ will be much larger than that of the magnetic moment M. Therefore, increasing the thickness is not a good way to reduce the measurement error in λ. Usually, the thickness $2d$ of the film approximately equals its penetration depth λ (typically, $2d \approx$ 50-500 nm). It is concluded that α should not be reduced by increasing $3\pi Nd$.

Therefore, we consider the alterative; decreasing the factor $2l\theta\varphi$. Recall that θ is the angle between the superconducting film surface and the magnetic field direction, and ϕ is the angle between the superconducting film surface and the axis of the measuring coil. Both angles are difficult to minimize without special devices. Typically, $\theta\approx\phi\approx 3^{\circ}$. Consequently, to reduce $2l\theta\varphi$, we must reduce l, the length of the film along the direction of the magnetic field.

Reducing l was difficult in the 1950s, but is easily accomplished with modern microfabrication technology. First, we fabricate the desired pattern on a mask, as shown in Fig. 1. The dimensions of each rectangle in Fig. 1 are 50 μm × 5 μm (where, the length normal to the direction of the magnetic field is 50 μm, and the length parallel to the direction of the field is 5 μm). Adjacent rectangles are separated by 3-5 μm gaps. Second, a photoresist layer is coated on the superconducting film; Third, the mask pattern is copied onto the photoresist by a photolithography process. Finally, the superconducting film uncovered by the photoresist is removed by physical or chemical etching technologies. Note that the film between adjacent rectangles must be etched completely, and that no superconducting connection between these small rectangles can exist below the critical temperature T_C. The completed superconducting film was then divided into a number of smaller similar rectangles, each with length l approximately 5 µm along the magnetic field direction

Now, substituting l = 5 µm and d = 150 µm into α, we obtain $\alpha = (1 + 7\theta\varphi)$. θ and φ are experimentally controllable to within ±3°, ensuring a satisfyingly small correction factor α (below 1.02).

4 Estimation of the Experimental Precision

The measurement accuracy of the penetration depth depends on both the correcting factor and the measurement accuracy of the specimen's magnetic moment. The latter is estimated as follows:

In experiment by Lock, the susceptibility of the specimens was measured by a galvanometer, which has relatively low accuracy. More recently, the magnetic moments of samples have been measured by the SQUID magnetometer, which resolves the magnetic moment to approximately 1 × 10^{-11} A·m^2.

Suppose that a substrate of area (10 × 10) mm^2 is coated with a 300 nm- thick superconducting film(i.e., 2d = 300 nm). The film is divided into a number of small rectangles as shown in Fig. 1. Each small rectangle is 50.0 µm long and 5.00 µm wide, and separated from its neighbours by 3 µm. Thus, the substrate is subdivided into 2.35×10^5 rectangles, each of volume 75 µm^3, covering a total volume V of 1.76 × 10^{-11} m^3. The magnetic induction of the field is 30 Gs (i.e., the magnetic intensity H = 2.38 × 10^3 A·m^{-1}). As the penetration depth of the specimen is varied, its magnetic moment M varies as:

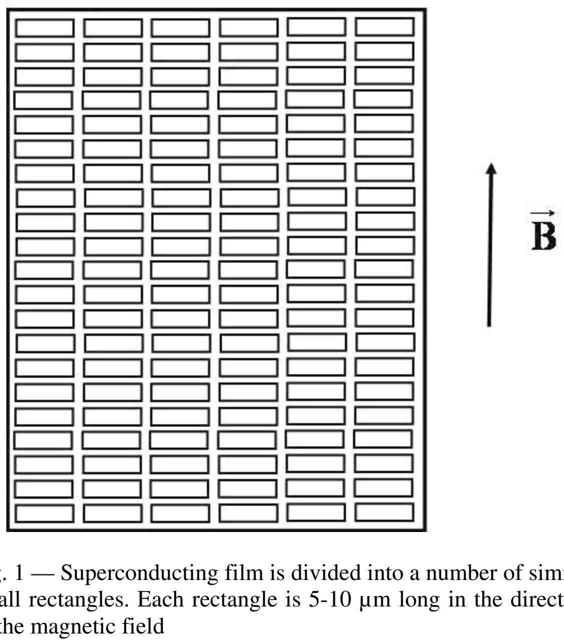

Fig. 1 — Superconducting film is divided into a number of similar small rectangles. Each rectangle is 5-10 µm long in the direction of the magnetic field

$$M = \chi VH \qquad \ldots(3)$$

where χ is determined by Eq. (1). The estimated magnetic moments are listed in Table 1.

In Table 1, the magnetic moment ranges from 592 × 10^{-11} A·m^2 to 2799 × 10^{-11} A·m^2, approximately 1000 times larger than the measurement accuracy δ of the SQUID magnetometer. In other words, the magnetic moment of the specimen can be measured to within ±0.1%–0.2%. For example, as the penetration depth λ of the sample increases from 100 nm to 101 nm, its magnetic moment changes from 1661 × 10^{-11} A·m^2 to 1643 × 10^{-11} A·m^2, giving a gradient $\Delta M/\Delta\lambda$ = 18 × 10^{-11} A·m^2/nm. Although the penetration depth λ changes by only 0.1 nm as the temperature increases, the magnetic moment alters by 1.8 × 10^{-11} A·m^2, which is measurable by the SQUID magnetometer.

When determining the absolute value of the penetration depth λ, we must consider not only the measurement accuracy δ of the magnetometer, but also the correcting factor α. The experimental error ΔM in the magnetic moment is contributed by two sources; the measurement accuracy δ (approximately 0.1%–0.2%) and the correcting factor α (2%-3%). As the experimental error introduced by the measurement accuracy δ is an order of magnitude smaller than that introduced by α, it can be neglected, and we need only compute the error introduced by the correcting factor α.

From Eq. (3), we obtain the following:

$$\frac{dM}{M} = \frac{d\chi}{\chi} \qquad \ldots(4)$$

Table 1 — With the penetration depth of the specimen taking different values, the magnetic moment M of the specimen is estimated. The unit of the moment is the minimum measurable moment δ=$10^{-11}A.m^2$

λ(nm)	50	70	90	110	130	150	170	190	210
M ($10^{-11}A.m^2$)	2799	2287	1848	1494	1215	999	830	697	592

and from Eq. (1), we get:

$$d\chi = \left[\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - \left(\operatorname{sech}\left(\frac{d}{\lambda}\right)\right)^2\right]\frac{d\lambda}{\lambda} \qquad \ldots(5)$$

Therefore, we have:

$$\frac{dM}{M} = \frac{d\chi}{\chi} = \left[\frac{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - \left(\operatorname{sech}\left(\frac{d}{\lambda}\right)\right)^2}{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - 1}\right]\frac{d\lambda}{\lambda} \qquad \ldots(6)$$

or, by rearranging,

$$\frac{d\lambda}{\lambda} = \left[\frac{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - 1}{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - \left(\operatorname{sech}\left(\frac{d}{\lambda}\right)\right)^2}\right]\frac{dM}{M} = -F\left(\frac{\lambda}{d}\right)\frac{dM}{M} \qquad \ldots(7)$$

where $F\left(\frac{\lambda}{d}\right)$ is the ratio of $-\frac{d\lambda}{\lambda}$ to $\frac{dM}{M}$, defined as follows:

$$F\left(\frac{\lambda}{d}\right) = -\frac{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - 1}{\frac{\lambda}{d}\tanh\left(\frac{d}{\lambda}\right) - \left(\operatorname{sech}\left(\frac{d}{\lambda}\right)\right)^2} \qquad \ldots(8)$$

The dependence of $F\left(\frac{\lambda}{d}\right)$ on $\frac{\lambda}{d}$ is plotted in Fig. 2. Obviously, $F\left(\frac{\lambda}{d}\right) > 0$.

For $\alpha = \left(1 + \frac{2l\theta\varphi}{3\pi Nd}\right) > 1$, the measured magnetic moment of the sample is overestimated; thus, $\frac{dM}{M} > 0$. However, since $F\left(\frac{\lambda}{d}\right) > 0$, $\frac{d\lambda}{\lambda} < 0$ from Eq. (7), which means that the measured penetration depth λ is smaller than the true λ.

Figure 2 shows that if $\frac{\lambda}{d} << 1$, $F\left(\frac{\lambda}{d}\right) >> 1$, i.e. $\left|\frac{d\lambda}{\lambda}\right| >> \left|\frac{dM}{M}\right|$, which means that the measurement error in the λ would be much larger than that of the magnetic moment M if the superconducting film thickness $2d >> \lambda$. We should avoid such a situation. That is the reason why we cannot reduce the correcting factor α by increasing the film thickness $2d$.

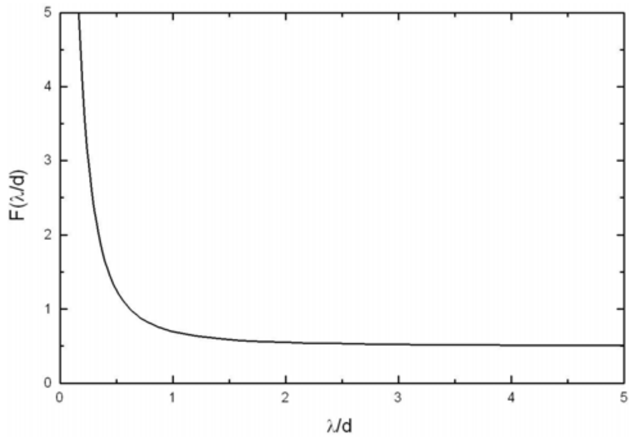

Fig. 2 — Dependence of $F(\lambda/d)$ on the (λ/d)

But, Fig. 2 also shows that $F\left(\frac{\lambda}{d}\right) < 1$ if $\frac{\lambda}{d} > 0.625$. It means that for the experiment in which the penetration depth $\lambda(0)$ of the superconducting film approximates the thickness $2d$, the measurement error in the absolute value of the penetration depth λ should not exceed that of the magnetic moment. Consequently, if $\frac{\Delta M}{M} < 3\%$, the measurement error $\left|\frac{\Delta\lambda}{\lambda}\right|$ should also be less than 3%. So, the penetration depth λ can be measured to a satisfactory level of accuracy as long as the thickness $2d$ of the superconducting film approximates its penetration depth $\lambda(0)$.

5 Conclusions

According to the analysis, our experimental scheme measures the absolute penetration depths of superconducting films to rather high accuracy. Furthermore, this method can be standardized, eliminating the need to determine absolute values of penetration depths and their temperature dependences by theoretical model fitting.

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