



# Exploiting Dynamic Programming in Optimizing Reliability-Centered Maintenance: Case Study of Medium-Sized Aluminum Manufacturing Plant

Safieh Almahmoud<sup>1</sup>, Mohammad Alkhedher<sup>2\*</sup> & Abedallah Al Kader<sup>3</sup>

<sup>1</sup>Mechanical Engineering, Khalifa University, Abu Dhabi, UAE

<sup>2</sup>Mechanical Engineering Department, Abu Dhabi University, Abu Dhabi, UAE

<sup>3</sup>Department of Integrated Systems Engineering, The Ohio State University, Ohio, USA

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Maintenance Scheduling (MS) is one of the most persistent issues that might arise in a manufacturing facility. It is crucial to do routine maintenance on machinery in order to avoid unplanned breakdowns. This type of failure could cause costly manufacturing process interruptions. Numerous strategies have been developed in an effort to deal with MS. But because each system has specific requirements and limitations, this is a particularly challenging problem. The scheduling of maintenance for machine units at a manufacturing facility that produces aluminum in moderate amounts is explored in this study using a dynamic programming approach. The Maintenance Scheduling model is put into practice using a Reliability-Centered Maintenance (RCM) strategy after an investigation of the architecture and infrastructure of the plant. By maintaining machines at acceptable dependability values and minimizing maintenance costs, this method is created to optimize the maintenance schedule. Here, factors like reliability and failure rate that affect the MS problem are discussed and investigated. Applying the model to a situation that represents the aluminum manufacturing allows it to be tested in a variety of different circumstances. The results of applying the model to the test cases are given, followed by a discussion of the results. The results obtained are reasonable and show that the dynamic programming strategy is a successful way to fix the MS issue that the manufacturing plant's machines are experiencing.

**Keywords:** Maintenance planning, Recursive model, Reliability-focused, Time-varying schedule

## Introduction

Regular preventive maintenance on the machine parts is crucial to prevent the issue of unexpected machinery failure in a production facility. This is because a machine malfunction could stop production and create an expensive backlog of work. A manufacturing facility's preventive maintenance schedule should be optimized to extend machine lifespan, improve machine dependability, and boost plant productivity by cutting maintenance costs. Maintenance scheduling is a complex optimization problem that has been an interesting subject of research over the past 50 years.<sup>1,2</sup> Classical mathematics and calculus cannot solve optimization problems because they involve several variables and limitations. In order to solve this type of complex problems, mathematical programming approaches, which are based on the large computational capability of modern computers, are used.<sup>3</sup> These approaches are based on retrieving an extreme value (minimum or

maximum) of a function to find the optimal solution from a wide range of solutions.<sup>4</sup> Various mathematical programming approaches have been applied to solve the MS optimization problem. The main mathematical approaches are the integer programming, branch-and-bound, dynamic programming, and benders decomposition.

Integer programming is a linear programming matter with non-negative integer variables. A 0-1 integer linear programming approach has been proposed by different researchers to schedule outages of plants.<sup>5</sup> This method was used to specify the time of outages as well as the amount of refueling in a way that satisfies the demand and minimizes the production cost during a period of time.<sup>6,7</sup> The approach was successful enough to win the first prize of EURO/ROADEF 2010 contest.

Another optimization method is the dynamic programming technique, which divides a large problem into a number of smaller, simpler problems.<sup>8,9</sup> This method is used to resolve issues where there are overlapping subproblems and an ideal substructure. The main feature which makes it an

\*Author for Correspondence  
E-Mail: mohammad.alkhedher@adu.ac.ae

effective approach for the optimization problem is the multistage nature of the solution process. Models usually include several states that are divided into a number of stages. The solution is achieved by moving sequentially from one stage to the next.

Korpijärvi, and Kortelainen<sup>10</sup>, have optimized the maintenance strategy of a turbine system using a dynamic programming model.<sup>9</sup> The strategy decides whether an overhauling or a reinvestment activity should be made on the component during a period of time. The dynamic programming method was implemented based on the reliability-centered maintenance approach to minimize the failure costs of components. When a component's reliability falls below the acceptable value, an action of either overhaul or reinvestment should be made. If the failure costs exceed the investment cost, a reinvestment of a new component should be made. However, there are situations when revamping a component is a more cost-effective option than reinvesting. No action, overhaul, or reinvestment are the three main options available in this model. It has been demonstrated that using this strategy to choose the best course of action for the electrical system's components throughout the scheduling period is efficient.

### Materials & Methods

From literature, it can be seen that there are many different methods which can be used to decide the maintenance schedule (MS). However, determining the most appropriate technique for usage depends on the nature and complexity of the maintenance problem. This paper reflects a test problem of scheduling and planning the maintenance of the machines at Reem Emirates Aluminum (REA) plant located in Abu Dhabi, UAE. The problem in hand is not too big, has discrete nature, and would benefit from comparing multiple alternatives. Therefore, the dynamic programming method would be a feasible approach to use.<sup>11,12</sup>

The test problem uses the dynamic programming model presented by Hedges.<sup>12</sup> The model utilizes an RCM approach. This type of preventive maintenance takes both the equipment's current condition (failure rate) and the analysis of needs and priorities into consideration, to determine a prioritization of the maintenance tasks. The goal of the model is to determine the optimized maintenance schedule that will provide the desired reliability at the lowest cost.

REA is an aluminum curtain wall factory that contains 95 different machine units in the plant. For the aim of simplification, we will concentrate on the maintenance of the SBZ 140 CNC machine. However, the model can be applied to all other machines of REA. There are 4 units of the SBZ 140 CNC machine in REA's plant. The current approach of maintenance used in the plant is Corrective Maintenance (CM), which means that a machine usually is not maintained until a failure occurs. Although the maintenance team of REA tries to inspect the machine regularly and change the major parts on a regular basis in order to reduce the number of failures in the machine, it has resulted in the machine being either under or over maintained. Since REA's MS problem has a discrete nature and is not too large, and since comparing possible options has a great benefit in deciding the optimal MS, the dynamic programming approach is a very suitable technique for REA's current MS problem.

The first step before developing the model at REA is to initiate planned shutdowns during the year where maintenance activities can be performed. The shutdowns are important to eliminate the frequent interruptions of production due to sudden maintenance activities. Of course, shutdowns shall not be utilized to maintain all the machines. Only the machines that are in need for maintenance shall be maintained during the shutdown time. This is because machine's over-maintenance will increase the cost, which in turn will reduce the plant efficiency. Moreover, under-maintenance of the machine increases the risk of failure. The model will result in an optimized maintenance schedule to help REA decide which shutdowns to utilize for performing preventive maintenance activities. The maintenance carried out shall keep the machine at a desirable reliability and the costs as low as possible.

### Theoretical consideration

#### Model Assumptions

To achieve the model-designated goals, the following assumptions are implemented:

- Maintenance processes done are proper, appropriate and do not decrease the reliability of the unit.
- The likelihood of failure increases exponentially with passing time (aging).
- Proposed shutdowns dates are fixed.

- All the four units of the SBZ 140 machine have the same failure rate.
- During the shutdowns, spare parts and maintenance team labour are always available (both internal and external team).
- The benefit created by each machine unit is very complex to measure.
- The machine condition after maintenance is “as good as new”.

**Model Parameters**

The model inputs and outputs are shown in Table 1.

**Reliability and Aging**

One of the key elements influencing the frequency of maintenance required is the machine age.<sup>13</sup> As the machine ages, its reliability decreases and it becomes in a higher need for maintenance because of the aging of its components. One way of determining the machine reliability is by knowing the deterioration rate of that machine. The rate of deterioration indicates how a unit improves or degrades with age.<sup>14</sup> A failure rate function is usually used for measuring the deterioration (aging) of the machine. Failure rate is the frequency with which a machine unit fails during a specific time period. The failure history data of the SBZ 140 is analyzed over 4 machine-years of service. As summarized in Table 2, the number of major failures during this period is 13 failures. Thus, average failure rate ( $\lambda$ ) for this interval is 3.25 failures per machine annually.

Poisson distributions are usually used to find the probability of failure function.<sup>15</sup> By knowing the

machine failure rate ( $\lambda$ ), failure probability over time can be computed as:

$$P(X > 0) = 1 - e^{-\lambda t},$$

where, t is the time period. (1a)

At REA, the SBZ 140 CNC machine has a rate of failure of 3.25 failures annually. Therefore, the failure probability is:

$$P(X > 0) = 1 - e^{-3.25t},$$

where, t is expressed in years. (1b)

Equation (1b) is used to plot the graphs shown in Fig. 1 and Fig. 2, which illustrate the relationship between failure probability and time. As seen in Fig. 1, the probability of failure is exponentially distributed over the long run. However, Fig. 2 shows that over the short run (6 months or less), The graph shows a nearly linear relationship between time and the likelihood of failure. Since the time periods between the shutdowns at REA plant rarely exceeds 6 months, this linear relationship can be used to determine the decrease in reliability between shutdowns for the machine units (decay rate). The decay rate in this case is 11.158, which is the slope of the line in Fig. 2.

**Maintenance Cost**

In the calculation procedure of the optimized MS, the maintenance cost is a very significant variable that needs to be considered. By demonstrating the cost

Table 1 — Model inputs and outputs

Inputs	Outputs
Machine Age	Approximate reliability for alternative schedules
Starting reliability	Maintenance costs of the alternative schedules
Maintenance costs	The optimal path (Schedule)
Time periods between shutdowns	Approximate reliability for the optimal schedule
Probability of failure after maintenance	Maintenance costs of the optimal schedule
Change in reliability over time	
MinimumDesired reliability	

Table 2 — Failure rate of SBZ 140 CNC machine

Sample size (Machine years)	No. of Failures	Failures per machine-year
4 machine-years	13 failures	3.25 failures

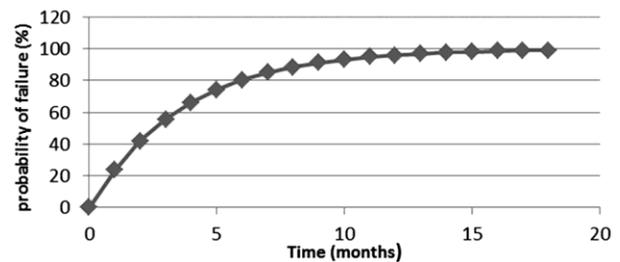


Fig. 1 — Relationship over long runs between failure probability and time

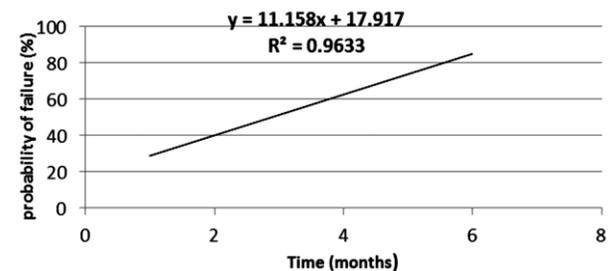


Fig. 2 — Relationship over short runs between failure probability and time

correlation between maintaining the equipment too early or too late, the total maintenance cost over time is estimated.<sup>16</sup> The total maintenance cost expected over time is represented in Fig. 3. It shows the maintenance investment lost by maintaining the SBZ 140 machine too early. It also shows how the expected maintenance cost increases with time as leaving the machine with no maintenance for a long time will dramatically increase the maintenance cost associated with it. This figure will be used in this model to find the total maintenance cost.

The cost values in the model do not represent the real maintenance costs because the profit gained from using the machine is not considered. The only thing considered here is the salvage value of the machine for the purpose of maximizing the reliability. Therefore, it is only used to evaluate the benefit of having a greater reliability (and its related effect on the salvage value) against the maintenance process expenses needed to have this reliability.<sup>17,18</sup>

**States and Stages**

In this dynamic programming model, the *state* is the probability of failure of the machine which is different from one state to another. The *stages* are the planned shutdowns in the plant. At the start of each stage (shutdown), a decision should be made. It is either to carry out a maintenance activity during the shutdown or not. Each decision will affect the machine reliability (probability of not failing) and the maintenance cost differently.

**The Recursion**

The main feature that makes the dynamic programming an effective approach for the optimization problems is the multistage nature of the solution process.<sup>19,20</sup> It is usually constructed using a recursion. The recursion breaks the problem into smaller subproblems, and it usually involves a function calling itself (recursive function). The

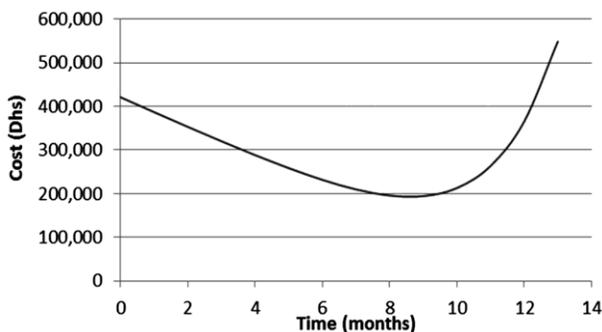


Fig. 3 — Total maintenance expenses as a function of time since the last maintenance

recursive function used in this model was presented by Hedges<sup>12</sup> as shown below.

**For  $p = P = 5$ :**

$$F_p(r) = \min \begin{cases} \text{Maintain: } M_r - C_y(u - d * t_p) + C_y(1 - (u - d * t_p)) \\ \text{Don't maintain: } -C_y(r - d * t_p) + C_y(1 - (r - d * t_p)) \end{cases} \dots (2)$$

**For  $p < P$ :**

$$F_p(r) = \min \begin{cases} \text{Maintain: } M_r + F_{p+1}(u - d * t_p) \\ \text{Don't maintain: } F_{p+1}(r - d * t_p) \end{cases} \dots (3)$$

where,

$F_p(r)$  = Minimum cost of the machine maintenance during shutdown  $p, p+1, \dots, P$

$r$  = Reliability

$d$  = Decay rate (decrease in reliability)

$S$  = Minimum acceptable reliability

$t_p$  = Time period (in months) to the next shutdown

$p$

$U$  = Machine reliability just after maintenance

$M_r$  = Cost of maintaining a machine with reliability

$r$

$C_y$  = Salvage value of machine at the end of scheduling period

$d, t_p, U, r, M_r, C_y, S \geq 0, r \geq S$ , initiated at  $F_p(r)$  and reverse the process to  $F_1$  (user defined  $r$ ).

“MATLAB” software is used to develop the code originally presented by Hedges.<sup>12</sup> It is modified and improved to fit the current application of the SBZ 140 CNC machine, to generate the optimal preventive maintenance schedule.

**Model Results and Discussion**

The proposed model is applied to a sample situation of REA SBZ 140 CNC machine. The equipment is seven years old and was recently serviced one month ago. The probability of not failing (reliability) directly after maintenance is assumed to be 99.8% as the unit becomes in the “as good as new” state after carrying out maintenance. The reduction in reliability per month (decay rate) is found to be 11.158%. Thus, the starting reliability of machine at the time of running the model is  $(99.8 - (11.158 \times 1))$  which equals 88.7%. The desired minimum reliability of the machine is 53%. The planned shutdown intervals ( $t_p$ ) used in the model sample and the optimal solution of maintenance decisions that has resulted after running the model are shown in Table 3.

Table 3 — Planned shutdown intervals and optimal solution

Shutdowns	1	2	3	4	5	End state	
Months in-between ( $t_p$ )	2	4	1	3	1	Months until nearest year end	
Solution Decision	M	M	D	M	D	Ending Salvage worth	

“M” indicates that the shutdown should be utilized for carrying out maintenance, while “D” indicates that maintenance is not recommended during that shutdown.

There are five different shutdowns in this example; therefore, the problem of MS will have 5 stages. The model's graphical representation is displayed in Fig. 4. There are two possible decisions in each stage. The first option is to maintain the machine. In this option, maintenance charge applies, but the machine reliability will increase. The second option is not to maintain the machine. In this option, the reliability will decrease and the cost of the next maintenance procedure may increase. In Fig. 4 the effect of each maintenance decision during each stage (planned shutdown) on the state (machine reliability) is shown. The numbers in the figure shows the reliability. The right and up arrows represent a “Do maintenance” activity, while the down arrow represents a “Do not do maintenance” activity. The optimal maintenance path [M, M, D, M, D] is clearly shown in the figure.

To determine the model's sensitivity to various parameters, the recursion is run using different values of the parameters for three different shutdown schedules as shown in Table 4. In the testing of each parameter, the values of all other parameters remain fixed. The results obtained are shown in the next subsections.

**Impact of Minimum Desired Reliability**

A range of different values of the minimum desired reliability are used to run the recursion. The resulted maintenance schedules and costs for the 3 suggested shutdown schedules are shown in Table 5.

Comparing the outputs, it can be seen that the various values of the minimum desired reliability will affect the resulted optimal maintenance schedule. It can also be concluded that the higher the reliability required, the more shutdowns should be utilized and the more expensive is the maintenance. In schedule 3, shown in Table 5, there are some shutdowns which are relatively very far apart such as the third one (7 months), and since the SBZ 140 decay rate is relatively high (11.158%), the reliability until this shutdown will highly decrease. Thus, any required

Table 4 — Shutdown schedules

Shutdown schedule 1					
Shutdowns	1	2	3	4	5
Months in-between	2	4	1	3	1
Shutdown schedule 2					
Shutdowns	1	2	3	4	5
Months in-between	3	2	1	4	2
Shutdown schedule 3					
Shutdowns	1	5	4	3	3
Months in-between	5	3	7	2	4

Table 5 — Model outputs for shutdown schedules

	Minimum Desired Probability of not failing (%)	Maintenance Schedule	Cost (AED)
schedule 1	55.17 to 100	Not achievable	—
	44.1 to 55.16	[M,M,D,M,D]	585,710
	21.7 to 44	[M,D,M,D,M]	415,870
schedule 2	21.6 or less	[D,M,D,D,M]	5,830.3
	55.17 to 100	Not achievable	—
	32.9 to 55.16	[ M,D,M,M,D]	584,510
schedule 3	21.7 to 32.8	[ D,M,D,M,D]	172,610
	21.7 to 100	Not achievable	—
	21.6 or less	[ M,M,M,D,M]	841,460

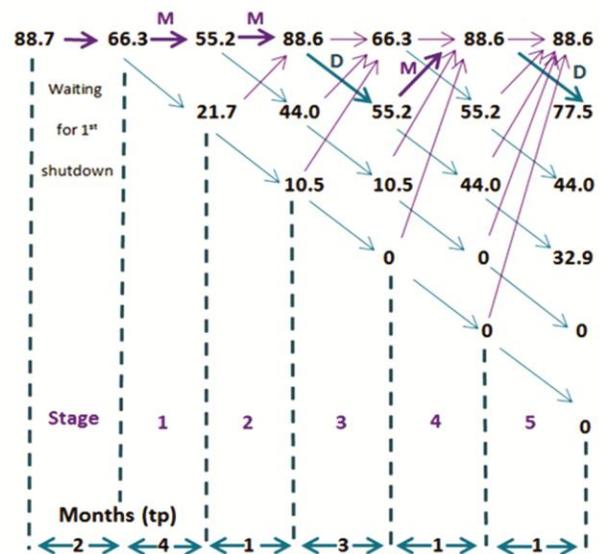


Fig. 4 — Graphical optimal path

reliability value equal or above the 21.7% is not achievable in this case.

**Impact of Machine Age**

A range of different machine age values are used to run the recursion. Maintenance schedules and costs resulting from running the model with these various

Table 6 — Model output for shutdown schedules 1 and 2, considering 55.16% minimum reliability at different machine ages.

	Age (years)	Maintenance schedule at 55.16% Minimum Reliability	Estimated Cost (AED)
Schedule 1	1	[ M,M,D,M,D]	329,420
	3	[ M,M,D,M,D]	414,850
	5	[ M,M,D,M,D]	500,280
	7	[ M,M,D,M,D]	585,710
	9	[ M,M,D,M,D]	671,140
	11	[ M,M,D,M,D]	756,570
Schedule 2	13	[ M,M,D,M,D]	842,000
	1	[ M,D,M,M,M]	193,540
	3	[ M,D,M,M,M]	348,330
	5	[ M,D,M,M,D]	499,080
	7	[ M,D,M,M,D]	584,510
	9	[ M,D,M,M,D]	669,940
	11	[ M,D,M,M,D]	755,370
	13	[ M,D,M,M,D]	840,800

age values, using shutdown schedules 1 and 2 are presented in Table 6.

From Table 6, it can be concluded that the machine age does not affect the maintenance schedule that much. It has only a small effect on the schedule. However, the age has a major effect on the maintenance cost because the salvage value is directly influenced by the machine age. As the machine age increases, the maintenance cost will also increase.

**Impact of Failure Rate**

The failure rate of the SBZ 140 machine is calculated based on the failure history data of 4 machine-years (3.25 failures annually). More data can be obtained over time, and the sample size of the study can be enlarged; therefore, a more accurate value of the failure rate can be recalculated. For the sake of comparison, a different failure rate value of 1.9 failures per year is assumed. The decay rate which corresponds to this value is 9.2904%. The results of running the model with this new value are shown in Table 7.

Comparing the results of Table 7 and Table 5, it can be clearly noticed that the optimal maintenance schedule was not affected by the change in the failure rate. However, the new smaller failure rate allowed having higher values of minimum desired reliability.

**Impact of Maintenance Cost**

The cost graph used to run the dynamic programming model was presented earlier in Fig. 3. For the sake of comparison, a different cost graph,

Table 7 — Model output for shutdown schedule 1, using a failure rate of 1.9 failures per year

Minimum Probability of not failing desired (%)	Maintenance Schedule	Cost (AED)
62.7 to 100	Not achievable	—
53.4 to 62.6	[ M,M,D,M,D]	538,850
34.8 to 53.3	[ M,D,M,D,M]	392,440

Table 8 — Comparison of model output using Fig. 3 (takes over-maintenance cost into consideration) and Fig. 5 (only actual maintenance costs are considered)

	Minimum Desired Probability of not failing (%)	Maintenance Schedule
Model run using Fig.3 (over-maintenance cost taken into consideration)	55.17 to 100 44.1 to 55.16 21.7 to 44	Not achievable [M,M,D,M,D] [M,D,M,D,M]
Model run using Fig.5 (only actual maintenance costs are considered)	55.17 to 100 1 to 55.16	Not achievable [M,M,M,M,M]

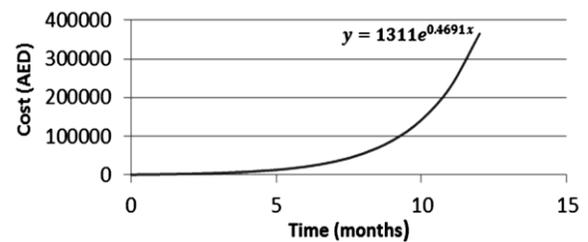


Fig. 5 — Estimated maintenance costs based on the interval since the last maintenance

which only represents the actual maintenance cost and does not take over-maintenance into consideration, is drawn in Fig. 5 and used to run the model.

The results of running the model in the two cases are shown in Table 8.

Comparing the results in Table 8, it can be seen that running the model using the figure which does not take over-maintenance into consideration has resulted in recommending two extra maintenance processes. Therefore, it can be concluded that taking into consideration the cost of over-maintenance can significantly optimize the maintenance schedule by avoiding the performance of maintenance too early or too late.

It was also found that the higher the reliability required, the more shutdowns that should be utilized. However, sometimes the required reliability value may not be achievable as the shutdowns may be set very far apart, in which case the reliability may fall below the minimum desired value.

## Conclusion

This study considers the problem of maintenance scheduling (MS) for an aluminum manufacturing plant utilizing a dynamic programming model to find an optimal preventive maintenance schedule. Results have shown that the dynamic programming method can give reasonable quality solutions. It has been also found that the higher the reliability required, the more shutdowns that should be utilized. Moreover, it can be concluded that taking costs of over-maintenance into consideration would significantly optimize the maintenance schedule by avoiding the performance of maintenance too early or too late. Thus, executing the model would benefit the plants by keeping the machines at acceptable reliability values to reduce the risk of sudden machine failures.

However, sometimes the required reliability value may not be achieved as the shutdowns may be set very far apart, in which case the reliability may fall below the minimum desired value. Also, maintenance data for all machines at all times might be sometimes not easy to get. Some data might be either missing or not recorded by the factory.

Further research can be made to improve this model. To do so, other variables can be considered, such as the availability of external maintenance team, the type of maintenance activity,....etc. Also, for more accurate results, a data recording system shall be established at the factory where maintenance data are recorded at all times by the maintenance team using big data bases. Furthermore, the model can be tested in other applications and its algorithm can be developed or modified to fit other systems such as railways and aircrafts.

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